REV-01 2022/08
BSP/01/04

B.Sc. PHYSICS
SIXTH SEMESTER (SPECIAL REPEAT)
MATHEMATICAL PHYSICS-III
BSP-603 A
(Use separate answer scripts for Objective & Descriptive)

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. Find the value of $(1+i)^{100}$.
   a. $\cos 100\pi + i\sin 100\pi$
   b. $2^{100}(\cos 100\pi + i\sin 100\pi)$
   c. $2^{50}(\cos 100\pi + i\sin 100\pi)$
   d. $2^{50}(\cos 50\pi + i\sin 50\pi)$

2. $f(z) = \bar{z}$ is differentiable
   a. Only at z=1
   b. Only at z=0
   c. Everywhere
   d. Nowhere

3. If z is a complex variable, the value of $\int_{5}^{3i} \frac{dz}{z}$ is
   a. -0.511-1.57i
   b. 0.511+1.57i
   c. 0.511-1.57i
   d. -0.511+1.57i

4. If $f(z) = x + ay + i(bx + cy)$ is analytic, then
   a. a=b=c=1
   b. a=1 and c=-b
   c. b=1 and a=-c
   d. c=1 and a=-b

5. A point at which a function ceases to be analytic is called
   a. Singular point
   b. Non-singular point
   c. Regular point
   d. Non regular point

6. Which of the following is an "even" function of t?
   a. t^2
   b. $t^2 - 4t$
   c. $\sin(2t) + 3t$
   d. $t^3 + 6$

7. A "periodic function" is given by a function which
   a. has a period $T = 2\pi$
   b. satisfies $f(t+T) = f(t)$
   c. satisfies $f(t+T) = -f(t)$
   d. has a period $T = \pi$

8. What are Fourier coefficients?
   a. The terms that are present in a Fourier series
   b. The terms that are obtained through Fourier series
   c. The terms which consist of the Fourier series along with their sine or cosine values
   d. The terms which are of resemblance to Fourier transform in a Fourier series are called Fourier series coefficients

9. Choose the condition from below that is not a part of Dirichlet's conditions.

a. It is a periodic signal, if the function f(x) for the interval (-π,π)

b. It is bounded, if the function f(x) for the interval (-π,π)

c. It has only a finite number of discontinuous, if the function f(x) for the interval (-π,π)

d. It is single-valued, if the function f(x) for the interval (-π,π)

10. A function f(x) is called skew symmetric function if

a. $f(-x) = -f(x)$

b. $f(-x) = f(x)$

c. $f(-x) = -f(-x)$

d. $f(-x) = 0$

11. $\int_0^\infty \frac{\sin ax}{x} dx = ?$

a. $\frac{\pi}{2}$

b. 0

c. $\sqrt{\frac{\pi}{2}}$

d. $\frac{\sqrt{\pi}}{2}$

12. In the following function f(x) is known as

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-isx} F(s) ds$$

a. Fourier transform of f(s)

b. Fourier transform of f(x)

c. Fourier transform of F(x)

d. Inverse Fourier transform of F(s)

13. Fourier sine transform of $\frac{1}{x}$

a. $\sqrt{\frac{\pi}{2}}$

b. $\frac{\sqrt{\pi}}{2}$

c. 0

d. ∞

14. Convolution of two function f(x) and g(x) is defined as

a. $f(x)*g(x) = \int_{-\infty}^{\infty} f(u)g(x/u)du$

b. $f(x)*g(x) = \int_{-\infty}^{\infty} f(u)g(x+u)du$

c. $f(x)*g(x) = \int_{-\infty}^{\infty} f(u)g(xu)du$

d. $f(x)*g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du$

15. Fourier transform of f(t)=..........x Laplace transform of g(t).

a. $\frac{1}{\sqrt{2\pi}}$

b. $\frac{1}{\sqrt{2\pi}}$

c. $\frac{1}{\sqrt{\pi}}$

d. None of these

16. Consider the Laplace transform of $F(x)$ is $f(s)$, and $L[F(ax)] = (1/a)f(s/a)$ then this property is known as,

a. Linearity property

b. Change of scale property

c. First shifting property

d. None of above

17. Laplace inverse transform of $\frac{1}{s^2 - 7s + 12}$ is

a. $e^{4x} + e^{3x}$

b. $e^{4x} - e^{3x}$

c. $e^{-4x} - e^{3x}$

d. $e^{4x} - e^{-3x}$

18. If $L\{F(t)\} = \bar{f}(s)$, then $L\{tF(t)\}$ is

a. $\bar{f}'(s)$

b. $-\bar{f}'(s)$

c. $\bar{f}'(s) + \bar{f}(s)$

d. $s\bar{f}'(s) + \bar{f}(s)$

19. $L^{-1}\{\frac{1}{s^n}\}$ exist only when the value of n is

a. Negative integer

b. Positive integer

c. Zero

d. None of these

20. Inverse Laplace transform of $\frac{s}{s^2 + a^2}$ is

a. $\cos at$

b. $\cosh at$

c. $\sinh at$

d. $\sin at$

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PART-B : Descriptive

Time : 2 hrs. 40 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a) Discuss the linear property of Fourier transform. 5+5=10

   b) Prove that the Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transform.

2. a) Solve the following equation by Laplace transform 8+2=10

   $y''' - 2y'' + 5y' = 0$; $y = 0$, $y' = 1$ at t=0and $y = 1$ at $t = \frac{\pi}{8}$.

   b) Find the value of $L^{-1}\{\frac{1}{(s^2 + a^2)^2}\}$

3. a) If $F_c(s) = \frac{1}{2}\tan^{-1}(\frac{2}{s^2})$, find $f(x)$. 6+4=10

   b) Establish the relationship between Fourier and Laplace transforms.

4. a) Obtain Laplace transform of derivative for order 'n'. 6+4=10

   b) Find the value of

   (i) $\int_c \frac{z+4}{z^2+2z+5}\,dz$, where c is the circle $|z+1|=1$.

   (ii) $\oint_c \frac{2z^2+5}{(z+2)^3(z^2+4)}dz.$, where c is the square with the vertices at $1+i$, $2+i$, $2+2i$, $1+2i$.

5. a) If $2\cos\theta = x + \frac{1}{2}$ and $2\cos\phi = y + \frac{1}{y}$, then prove that 5+5=10

   $x^p.y^q + \frac{1}{x^p.y^q} = 2\cos(p\theta + q\theta)$.

   b) Test the analyticity of the function $w = \sin z$ and hence derive that $\frac{d}{dz}(\sin z) = coz$

6. a) If $x = \cos\theta + i\sin\theta$, $y = \cos\phi + i\sin\phi$, prove that 5+5=10

   $\frac{x-y}{x+y} = i\tan(\frac{\theta-\phi}{2})$.

b) Expand the function $f(x) = x \sin x$, as a Fourier series in the interval $-\pi < x < \pi$.

7. a) Write the Fourier constant to evaluate the Harmonic analysis. 3+2+5=10

b) Represent the following function by a Fourier sine

series: $f(t) = \begin{cases} t, & 0 < t \le \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < t \le \pi \end{cases}$

c) Show that $L[f(t)u(t-a)] = e^{-as} L[f(t+a)]$

8. Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier expression of $f(x)$. Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots$ 8+2=10

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