

**M.Sc. MATHEMATICS  
FOURTH SEMESTER  
GENERAL MATHEMATICS-II  
MSM-406**

Duration : 3 hrs.

Full Marks: 70

**[ PART-A: Objective ]**

Time : 20 min.

Marks : 20

Choose the correct answer from the following:

1X20=20

- Which of the following is not a group?
  - $(\mathbb{R}, \cdot)$
  - $(\mathbb{Z}, +)$
  - $(\mathbb{R}, +)$
  - None of these
- Which of the following is true?
  - Identity element of a group is always unique.
  - Inverse element of a group is always unique.
  - Both (a) and (b)
  - None of these
- Which of the following is an Abelian Group
  - The group of all  $2 \times 2$  invertible matrix w.r.t matrix multiplication.
  - The group  $\{1, -1, i, -i\}$  w.r.t multiplication.
  - Both (a) and (b).
  - None of these
- Let  $G$  be a group and  $H$  be a subgroup of  $G$ . If order of  $H$  is 6, then order of  $G$  can be
  - 2
  - 12
  - 15
  - None of these.
- Let  $G = \{0, 1, 2\}$ . For  $a, b \in G$ , a binary operation ' $*$ ' define on  $G$  as  $a * b = |a - b|$ . The  $(G, *)$  is
  - a group
  - an Abelian group
  - not a group
  - none of these
- The set of natural number  $\mathbb{N}$  is
  - bounded
  - bounded below but not bounded above
  - bounded above but not bounded below
  - not bounded
- The infimum and supremum of the set  $\{-1, -\frac{1}{2}, -\frac{1}{3}, \dots\}$  are respectively
  - 1 and 0
  - 0 and -1
  - Does not exist
  - None of these
- Consider the following statements:  
P: The set  $\mathbb{R}$  of real numbers is the neighbourhood of each of its points.  
Q: The set  $\mathbb{Q}$  of rational numbers is not the neighbourhood of each of its points.

- a. P true, Q false  
b. P false, Q true  
c. Both P and Q are true.  
d. Both P and Q are false.
9. Which of the following is/are not open set?  
a. The set of real numbers,  $\mathbb{R}$   
b. The set of natural numbers,  $\mathbb{N}$   
c. The null set  $\Phi$   
d. None of these
10. Consider the following statements:  
P: Every finite set of numbers is bounded.  
Q: Any interval in the real line  $\mathbb{R}$  is bounded.  
a. P true, Q false  
b. P false, Q true  
c. Both P and Q are true.  
d. Both P and Q are false.
11. Let  $G$  be the set of all  $2 \times 2$  square matrices. The identity element of  $(G, +)$  is  
a. 0  
b. 1  
c. Zero matrix  
d. Unit matrix
12. Which of the following is/are open sets?  
a.  $[1, \alpha]$   
b.  $(-\alpha, 0]$   
c.  $(0, 1)$   
d.  $[0, 1]$
13. If  $f(x) = x^2 \sin \frac{1}{x}$ ,  $x \neq 0$ , then the value of the function  $f(x)$  at  $x = 0$ , so that the function is continuous at  $x = 0$  is  
a. 0  
b. -1  
c. 1  
d. None of these
14. The set of points where the function  $f$  given by  $f(x) = |2x - 1| \sin x$  is differentiable is  
a.  $\mathbb{R}$   
b.  $\mathbb{R} - \left\{\frac{1}{2}\right\}$   
c.  $(0, \infty)$   
d. None of these
15. The function  $f(x) = \cot x$  is discontinuous on the set  
a.  $\{x = n\pi, n \in \mathbb{Z}\}$   
b.  $\{x = 2n\pi, n \in \mathbb{Z}\}$   
c.  $\left\{x = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}\right\}$   
d. None of these
16. The function  $f(x) = \frac{4-x^2}{4x-x^3}$  is  
a. Discontinuous at only one point at  $x = 0$ .  
b. Discontinuous at exactly two points  
c. Discontinuous at exactly three points  
d. None of these
17. If  $f(x) = \sqrt{25 - x^2}$ , then  $\lim_{x \rightarrow 2} \frac{f(x) - f(1)}{x - 1}$  is equal to  
a.  $\frac{1}{24}$   
b.  $\frac{1}{5}$   
c.  $-\sqrt{24}$   
d.  $\frac{1}{\sqrt{24}}$

18.

Consider function  $f(x) = \begin{cases} x, 0 \leq x \leq 1 \\ 2-x, 1 < x \leq 2 \end{cases}$

- a.  $f$  is continuous at  $x = 1$   
c.  $f$  is continuous at  $x = 0$

- b.  $f$  is not continuous at  $x = 1$   
d.  $f$  is not continuous at  $x = 0$

19. Consider the following statement:

P: Every continuous function is differentiable.

Q: Every differentiable function is continuous.

- a. P true, Q false  
c. Both P and Q are true

- b. P false, Q true  
d. Both P and Q are false

20. Which of the following function is not differentiable?

a.  $f(x) = |x|, \forall x \in \mathbb{R}$

b.  $f(x) = \sin x, \forall x \in \mathbb{R}$

c.  $f(x) = x^3 + 3, \forall x \in \mathbb{R}$

d. None of these

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**( PART-B : Descriptive )**

Time: 2 HRS 40 MINS

Marks : 50

[ Answer question no.(1) & any four (4) from the rest ]

1. a. Show that the set  $\{\pm 1, \pm i, \pm j, \pm k\}$ , where  $i^2 = j^2 = k^2 = -1$  and  $ij = -ji = k, jk = -kj = i, ki = -ik = j$  is a group w.r.t multiplication. Is this an Abelian group? 4+6=10
- b. Find the derivative if
- (i)  $y = \sec^{-1} \frac{\sqrt{x}+1}{\sqrt{x}-1} + \sin^{-1} \frac{\sqrt{x}-1}{\sqrt{x}+1}$
- (ii)  $\sin x = \frac{2t}{1+t^2}, \cos y = \frac{1-t^2}{1+t^2}$
2. Find infimum and supremum of the following sets. Which of the them are bounded? 2×5=10
- (i)  $\left\{ \frac{1}{n} : n \in \mathbf{N} \right\}$
- (ii)  $\left\{ -2, -\frac{3}{2}, -\frac{4}{2}, -\frac{5}{4}, \dots, -\frac{n+1}{n}, \dots \right\}$
- (iii)  $\{x \in \mathbb{R} : 1 < x < 2\}$
- (iv)  $\{1, 3, 5, 7, \dots\}$
- (v)  $\{1 + (-1)^n : n \in \mathbf{N}\}$
3. Find the derived set of the following sets: 3+3+2=10
- (i)  $\{x : 0 \leq x \leq 1\}$
- (ii)  $\left\{ \frac{1}{n} : n \in \mathbf{N} \right\}$
- (iii)  $\mathbf{Z}$ , the set of integers
- (iv)  $\mathbf{Q}$ , the set of rational numbers
4. Let  $G$  be an Abelian group. Prove or disprove that the following sets are subgroup of  $G$ : 5+5=10
- (i)  $H = \{x^2 : x \in G\}$
- (ii)  $H = \{x \in G : x^2 = e\}$ , where  $e$  is the identity of  $G$ .

5. Solve

5+5=10

a.  $\int \frac{dx}{x^3 - x^2 - x + 1}$

b.  $\int \frac{(x-1)e^x}{(x+1)^2} dx$

6. Find  $\frac{dy}{dx}$

2×5=10

(a)  $\sqrt{\frac{(x-2)(x-3)}{(x-4)(x-5)(x-6)}}$

(b)  $\sin x^{\cos x} + \cos \sqrt{1-x^2}$

7. Examine the continuity of the function

10

$$f(x) = \begin{cases} 3, & 0 \leq x \leq 1 \\ 4, & 1 < x < 3 \\ 5, & 3 \leq x \leq 10 \end{cases}$$

8. Find the values of  $a$  and  $b$  if the function  $f(x)$  defined by

10

$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$$

Is differentiable at  $x = 1$ .

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