REV-01
MSM/58/64

2022/07

M.Sc. MATHEMATICS
SECOND SEMESTER
COMPLEX ANALYSIS
MSM-203

Duration : 3 hrs. **Full Marks: 70**

(PART-A: Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: *1X20=20*

1. The value of i where i is the square root of -1 is
   a. $e^{\frac{\pi}{2}}$
   b. $e^{-\frac{\pi}{2}}$
   c. $e^{i\frac{\pi}{2}}$
   d. $e^{-i\frac{\pi}{2}}$

2. The principal value of $Log(i^{1/4})$ is
   a. $i\pi$
   b. $\frac{i\pi}{2}$
   c. $\frac{i\pi}{4}$
   d. $\frac{i\pi}{8}$

3. Consider the function $f(x) = x^2 + iy^2$ and $g(x) = x^2 + y^2 + ixy$ at $z = 0$ then
   a. f is analytic but not g
   b. g is analytic but not f
   c. Both the functions are analytic
   d. None is analytic

4. For $z \in C$ define $f(z) = \frac{e^z}{e^z - 1}$, then
   a. The only singularities of f are poles
   b. f has infinitely many poles in the imaginary axis
   c. Each pole of f is simple
   d. All of the above

5. The function $f(z) = |z|$ is
   a. differentiable
   b. Nowhere differentiable
   c. Differentiable at $z = 0$
   d. None

6. F is said to be entire if
   a. F is analytic on C
   b. F is regular on C
   c. ∞ is the only possible singularity
   d. All of the above

7. An isolated singularity which is neither a removable singularity nor a pole is called

a. Essential singularity
b. Essential removable singularity
c. Isolated removable singularity
d. None

8. The number $\sqrt{2}\ e^{i\pi}$ is

a. Rational Number
b. A transcendental number
c. An irrational number
d. An imaginary number

9. If $f(z) = u(x, y) + ixy$ is analytic then

a. $u(x, y) = x^2 - y^2$
b. $u(x, y) = \frac{1}{2}(x^2 + y^2)$
c. $u(x, y) = \frac{1}{2}(x^2 - y^2)$
d. $u(x, y) = x^2 + y^2$

10. The function $f : C \to C$ defined by $f(z) = |z|$ is

a. Analytic when $z \neq 0$
b. Analytic everywhre
c. Nowhere analytic
d. Analytic in $\{z : |z| > 1\}$

11. The unction $f(z) = Sin\left(\frac{1}{z}\right), z = 0$ is a

a. Removable singularities
b. Simple pole
c. Non-isolated singularity
d. Essential singularity

12. If $f(z) = z^3$ then it

a. has an essential singularity at $z = \infty$
b. has a pole of order 3 at $z = \infty$
c. has a pole of order 3 at $z = 0$
d. is analytic at $z = \infty$

13. $\int_C z^2 dz$ where C is the circle with centre O and radius 2 equal

a. $4\pi^2$
b. $2\pi i$
c. 4
d. 0

14. The value of the integral $\int_C \frac{1}{z-1} dz, C : |z| = 4$ is equal to

a. πi
b. $2\pi i$
c. $4\pi i$
d. 0

15. In the Laurentz series of $\frac{Sin\, z}{z^2}$ at $z = 0$ the co-efficient of $\frac{1}{z}$ is

a. zero
b. π
c. -1
d. 1

16. $f(z)=\frac{Sinz}{z^2}$ has

a. A removable singularity at $z=0$

b. A pole of order 1 at $z=0$

c. No singularities

d. An essential singularity at $z=0$

17. The residue of $e^{1/z}$ at $z=0$ is

a. 2

b. ∞

c. 1

d. -2

18. The residue of $\frac{Sinz}{z^8}$ at $z=0$ is

a. 0

b. $\frac{1}{7!}$

c. $-\frac{1}{7!}$

d. None

19. $\int_0^{1+i}(x^2-iy)dz$ along $y=x$ is

a. $-\frac{1}{6}(5-i)$

b. $\frac{1}{6}(5-i)$

c. $-\frac{1}{6}(5+i)$

d. $\frac{1}{6}(5+i)$

20. $\oint_C \frac{z^2+5}{z-3}dz$ where C is the circle $|z|=1$

a. 0

b. $2\pi i$

c. $-2\pi i$

d. πi

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(PART-B : Descriptive)

Time: 2 HRS 40 MINS . Marks : 50

[Answer question no.(1) & any four (4) from the rest]

1. a. Determine the analytic function whose real part is $e^{2x}(xCos2y - ySin2y)$ 5+5=10

   b. Determine the analytic function whose imaginary part is $Sinhx\,Coshy$

2. 5+5=10

   a. Find the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ in $z-$plane under the transformation $w = \frac{1}{z}$

   b. Show that the function $f(z) = \sqrt{|xy|}$ s not analytic at the origin, although Cauchy-Riemann conditions are satisfied.

3. 5+5=10 -

   a. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y) but are not harmonic conjugates

   b. Calculate $\oint_C \log z\, dz$ where C is the circle $|z - 2| = 3$

4. 5+5=10

   a. Calculate the integral $\int_0^{1+i} (x - y + ix^2)dz$ along the real axis from $z = 0$ to $z = 1$ and the along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$

   b. Evaluate $\frac{1}{2\pi i}\oint_C \frac{ze^z}{(z-a)^3}dz$ where C is the circle $|z| = a$

5. 5+5=10

   a. Evaluate $\oint_C \frac{Sin^6 z}{(z - \frac{\pi}{6})^3}dz$ where C is the circle $|z| = 1$

b. Deduce that, $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}.\frac{\partial u}{\partial r} + \frac{1}{r^2}.\frac{\partial^2 u}{\partial \theta^2} = 0$

6. a. Show that the function $e^{-2xy}.Sin(x^2 - y^2)$ is harmonic 5+5=10

b. Apply Residue theorem to evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)}dz$ where C is the circle $|z| = \frac{3}{2}$

7. a. Apply Residue theorem to evaluate $\oint_C \frac{2z^2+5}{(z+2)^2(z^2+4)}dz$ on the circle $C:|z|=3$ 5+5=10

b. Expand by Laurentz series $\frac{1}{z^2-4z+3}$ for the region $1<|z|<3$

8. a. Evaluate the following integral by using Residue theorem $\oint_C \frac{z}{(z-1)(z-2)^2}dz$ where C is the circle $|z-2| = \frac{1}{2}$ 5+5=10

b. Expand by Laurentz series $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region $|z|>4$

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