Write the following information in the first page of Answer Script before starting answer

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number______________________________

Course____________________________ Semester____________________________

Paper Code _________________Paper Title __________________________________

Type of Exam: ______________________________ (Regular/Back/Improvement)

Important Instruction for students:

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

M.Sc. MATHEMATICS
THIRD SEMESTER
FUNCTIONAL ANALYSIS
MSM-302

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. Which of the following is false?
- **a.** Every vector space is a metric space
- **b.** Every normed linear space is a metric space.
- **c.** Every complete subspace M of a normed linear space N is not closed
- **d.** R^n is a banach space w.r.t. any norm

2. Two normed linear space X and Y are said to behomeomorphic if the map $f: X \to Y$ is:
- **a.** One to one and onto
- **b.** f^{-1} is continuous
- **c.** f is continuous
- **d.** All

3. A linear map from a normed space X to Y is continuous iff T is?
- **a.** Bounded
- **b.** Convergent
- **c.** Continuous
- **d.** Complete

4. A linear map $T: X \to Y$ is continuous iff T is continuous at:
- **a.** Any point x
- **b.** $x = 1$
- **c.** $x = 0$
- **d.** None

5. Which of the following is false?
- **a.** The set $B(X, Y)$ of all bounded linear operators from X $into$ Y is a subspace of the space $L(X, Y)$
- **b.** A finite dimensional normed space is not a banach space
- **c.** All norms on finite dimensional linear space are equivalent
- **d.** None

6. Which of the following statement is/are true?
- **a.** Integral operators are bounded
- **b.** Differential operators are bounded
- **c.** Integral operators are unbounded
- **d.** None

7. Two norms $\|.\|_1$ and $\|.\|_2$ are said to be equivalent if $\exists\ m, M > 0$ $such\ that$:
- **a.** $m\|x\|_1 \le \|x\|_2 \le M\|x\|_1$
- **b.** $m\|x\|_1 \ge \|x\|_2 \le M\|x\|_2$
- **c.** $m\|x\|_1 \ge \|x\|_2 \le M\|x\|_2$
- **d.** $m\|x\|_1 \le \|x\|_2 \ge M\|x\|_2$

8. Which of the following statement is true?
- **a.** A real inner product space is conjugate symmetric
- **b.** A complex inner product space is linear in the second argument
- **c.** A complex inner product space is symmetric
- **d.** A real inner product space is linear in the first argument

9. Which of the following is/are true for adjoint operator?

a. It doesn't preserve addition
b. It is linear
c. It preserve multiplication
d. It is conjugate linear

10. A linear map T is said to be invertible if:

a. T is continuous
b. T^{-1} exists
c. Both (a) and (b)
d. None

11. Which of the following is true?

a. An inner product space is not a normed linear space
b. Any finite dimensional normed linear space is a banach space
c. The function space C[a,b] is not complete
d. None

12. The Riesz representation theorem is not true in an inner product space which is:

a. Complete
b. Not complete
c. Compact
d. None

13. Which of the following statement is false?

a. In a Hilbert space the norm induced by the inner product satisfies the parallelogram law.
b. In l_n^1 space, where $n > 1$ the parallelogram law not is true
c. In l_n^1space, where$n > 1$ the parallelogram law is true
d. A complete inner product space is called Hilbert space

14. Which of the following is true?

a. A closed unit sphere in a normed linear space is not convex
b. A open unit sphere in a normed linear space is not convex
c. If N is a normed linear space then $||.||: N \rightarrow R$ is continuous on N
d. None

15. Which of the following statements is true?

a. Any two norms in a finite dimensional space is always equivalent
b. An inner product space is not a normed space
c. Any two norms in an infinite dimensional space is always equivalent
d. None

16. The principle of uniform boundedness is also known as:

a. Closed graph theorem
b. Riesz theorem
c. Open mapping theorem
d. Banach-steinhaus theorem

17. Let M is the subset of a normed space N. Given that d(M)be the diameter of M. Then M is bounded iff:

a. d(M) is infinite
b. d(M) is bounded
c. d(M) is finite
d. d(M) is continuous

18. Find the false statement.

a. Hahn banach theorem is true for any norm space
b. The closed graph theorem gives notion of closedness of the graph of a LT between banach spaces
c. A norm in an inner product space is defined by $\|x\|^3 = \sqrt{(x,x)}$
d. A norm in an inner product space is defined by $\|x\| = \sqrt{(x,x)}$

19. Which of the following is incorrect?

a. The Hahn- Banach theorem deals with extension of Linear operator

b. The set of all eigen values of T is called the spectrum

c. The Hahn- Banach theorem deals with extension of Linear functional

d. The discrete metric space is not a normed space

20. What is the spectrum of the idempotent operator T on a Banach space?

a. {1,-1}

b. {0, -1}

c. {0,1}

d. None

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(PART-B : Descriptive)

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. Write the statements of open mapping theorem and closed graph theorem. State and prove Riesz-representation theorem. 2+8=10

2. **a.** Given that T is a linear operator such that T: $X \xrightarrow{onto} Y$, both X and Y are normed spaces. Prove that T^{-1} exists and and is a bounded linear operator iff $\exists a constant K > 0$ such that $||Tx||_Y \geq K||x||_Y$, $x \in X$. 5+5=10

b. Given $\|.\|_1$ and $\|.\|_2$ be two equivalent norms on a linear space X. Prove that $(X, \|.\|_1)$ is Banach iff $(X, \|.\|_2)$ is also Banach.

3. **a.** Let f be a linear functional on a normed space N. If f is continuous at $x_0 \in N$, prove that it must be continuous at every point of N. 4+6=10

b. State Hahn-Banach theorem. If N is a normed linear space and x_0 is a non- zero vector in N, then there exists a functional f_0 in N^* *such that* $f_0(x_0) = ||x_0||$ *and* $||f_0|| = 1$.

4. **a.** Given M be a closed linear subspace of a normed linear space N and let x_0 be a vector not in M. If d is the distance from x_0 to M, then there exists a functional $f_0 \in N^*$ such that $f_0(M) = 0$, $f_0(x_0) = d$ *and* $||f_0|| = 1$. 6+4=10

b. Prove that in the Hilbert space l_2, the set $\{e_1, e_2 \dots\dots e_n \dots\}$ where e_n is the sequence with 1 in the nth place and 0's elsewhere is a complete orthonormal set.

5. **a.** The mapping $\phi: H \to H^*$ defined by $\phi(y) = f_y$ where $f_y(x) = < x, y >$ for every $x \in H$. Prove that ϕ is an additive, one to one, onto isometry but not linear. 5+5=10

b. If x *and* y are any two vectors in a Hilbert space H, then prove the polarization identity.

6. **a.** State and prove Banach-Steinhaus Theorem. 6+4=10

b. Prove that an operator T on a finite dimensional Hilbert space H is singular if and only if there exists a non-zero vector x in H such that $Tx = 0$.

7. **a.** Define Hilbert space. Let x *and* y be two orthogonal vectors in a Hilbert space H, then prove that $||x + y||^2 = ||x - y||^2 = ||x||^2 + ||y||^2$. 3+3+4=10

Show that the space l^p *with* $p \neq 2$ is not an inner product space.

b. Derive Cauchy-Schwarz inequality from Bessel's inequality.

8. **a**. Define self adjoint operators. If A_1 *and* A_2 two are self adjoint operators on H, then prove that their product A_1A_2 is self adjoint if and only if they commute. 5+5=10

b. Let H be the given Hilbert space and T^* be the adjoint of the operator T. Prove that T^* is a bounded linear transformation and T determines T^* uniquely.

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