Write the following information in the first page of Answer Script before starting answer

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number______________________________

Course____________________________ Semester____________________________

Paper Code _________________Paper Title __________________________________

Type of Exam: ______________________________ (Regular/Back/Improvement)

Important Instruction for students:

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

B.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PHYSICS-II
BSP-301

Duration : 3 hrs. **Full Marks : 70**

PART-A : Objective

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. In a square matrix, each diagonal element is zero and $a_{ij} = -a_{ji}$. Then its matrix will be:

a. Symmetric
b. Skew-symmetric
c. Hermitian
d. Skew-hermitian

2. The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigen value is:

a. -1
b. 0
c. 2
d. 1

3. One of the eigen value of the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is 5, then the other two eigen values are:

a. 0 and 0
b. 1 and 1
c. 0 and 1
d. 1 and -1

4. The polynomial $2x^2 + x + 3$ in terms of Legendre's polynomial is:

a. $\frac{1}{3}[4P_2 - 3P_1 + 11P_0]$
b. $\frac{1}{3}[4P_2 + 3P_1 - 11P_0]$
c. $\frac{1}{3}[4P_2 + 3P_1 + 11P_0]$
d. $\frac{1}{3}[4P_2 - 3P_1 - 11P_0]$

5. The generating function of Legendre's polynomial $P_n(x)$ is:

a. $\sqrt{1 - 2xu + u^2}$
b. $\frac{1}{\sqrt{1 - 2xu + u^2}}$
c. $(x^2 - 1)^n$
d. $\frac{1}{1 - 2xu - u^2}$

6. The orthogonal property of Legendre's polynomial is:

a. $\int_{-1}^{1} P_m(x) P_n(x) dx = 1$, if $m \neq n$,
b. $\int_{-1}^{1} P_m(x) P_n(x) dx = 0$, if $m \neq n$

c. $\int_{-1}^{1} P_m(x)P_n(x)dx = \infty$, if $m \neq n$

d. None of these

7. If A and B are the matrices of same order such that $AB = A$ and $BA = B$, then A and B are

a. Nilpotent

b. Idempotent

c. Singular

d. Hermitian

8. If $P_n(x)$ and $Q_n(x)$ are two independent solution of Legendre equation then the general solution is:

a. $y = AP_n(x) + BQ_n(x)$,

b. $y = AP_n^2(x) + BQ_n(x)$

c. $y = AP_n^2(x) + BQ_n^2(x)$

d. None of these

9. If x=0 is a regular singular point of the differential equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ and m_1, m_2 are real and different roots then:

a. $y = c_1(y)_{m_1} + c_2(y)_{m_2}$

b. $y = c_1(y)_{m_1} + c_2(\frac{dy}{dm})_{m_1}$

c. $y = c_1(y)_{m_1}$

d. $y = c_1(y)_{m_1} + c_2(y)_{m_1}$

10. The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$

a. $z = -x^{-2}\sin(xy) + xf(x) + g(x)$

b. $z = -x^2\sin(xy) - yf(x) + g(x)$

c. $z = -y^2\sin(xy) + yf(x) + g(x)$

d. $z = -x^2\sin(xy) + yf(x) + g(x)$

11. A partial differential equation has:

a. One independent variable

b. Two or more independent variables

c. More than one dependent variable

d. Equal number of dependent and independent variables

12. The matrix *A* is defined as $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$. The eigenvalues of A^2 is:

a. -1, -9, -4

b. 1, 9, 4

c. -1, -3, 2

d. None of these

13. How many constants are required to make a 2nd order partial differential equation?

a. 3

b. 2

c. 1

d. 0

14. Complete solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$ is:

a. $z = f_1(y+2x) + xf_2(y+2x)$

b. $z = f_1(2y+x) + xf_2(2y+x)$

c. $z = f_1(y+x+2) + xf_2(y+x+2)$

d. $z = f_1(y-2x) + xf_2(y-2x)$

15. Which of the following is Lagrange's linear equation?

a. $Pp + Qq = R$

b. $Pp + Qq \neq R$

c. $Pp + Qq = 0$

d. None of these

16. What is the value of $\beta(z,1)$?

a. $\frac{1}{z}$

b. $\frac{1}{z+1}$

c. $\frac{1}{z(z+1)}$

d. $\frac{1}{z-1}$

17. $\frac{\Gamma(-1/2)}{\Gamma(1/2)}$

a. 2

b. -2

c. $\frac{1}{2}$

d. $-\frac{1}{2}$

18. If $1.3.5.......(2n-1) = \frac{2^n}{\sqrt{\pi}}\Gamma(p)$, then p is:

a. $n+\frac{2}{3}$

b. $n+\frac{1}{2}$

c. $n+\frac{1}{3}$

d. $\frac{n}{2}-1$

19. $\int_0^\infty e^{-t^2} dt = ?$

a. $\sqrt{\pi}$

b. $\frac{\sqrt{\pi}}{2}$

c. π

d. 0

20. The error function $erf(x)$ is written as:

a. $\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$

b. $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$

c. $\frac{\sqrt{\pi}}{2}\int_0^x e^{-t^2}\,dt$

d. $\frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}\,dx$

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PART-B : Descriptive

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in a rectangle in the xy-plane with $u(x,0)=0, u(x,b)=0, u(0,y)$ and $u(a,y)=f(y)$ parallel to y-axis. 10

2. Prove that the product of two matrices 4+6=10

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{and} \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} \text{is zero}$$

then θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

Find the inverse of following matrix by elementary row transformation

$$A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

3. Evaluate Beta function in terms of gamma function. 4+4+2=10

Prove that $1.3.5....(2n-1)=\frac{2^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}$

Show that $\beta(l,m)=\beta(m,l)$.

4. 5+5=10

i. Evaluate $\int_0^{\frac{\pi}{2}} sin^p\theta \, cos^q\theta \, d\theta$

ii. Evaluate $\int_0^{\frac{\pi}{2}} sin^{m-1}(2\theta)\, d\theta$

5. **i.** Using elementary transformations, reduce the following matrix to normal form and find its rank 6+4=10

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 1 & 2 \end{bmatrix}$$

ii. Verify whether the matrix $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$ is orthogonal.

6. Obtain the general solution of one dimensional wave equation by using the method of separation of variables. 5+5=10

Solve

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos\ x\cos 2y$$

7. 7+3=10

i. Using Frobenius methods solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-1)y = 0$.

ii. Solve the equation $\frac{d^2y}{dx^2} + x^2y = 0$

8. 4+5+1=10

Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, if $m \neq n$

Prove that

$$P_n(-\frac{1}{2}) = P_0(-\frac{1}{2})P_{2n}(\frac{1}{2}) + P_1(-\frac{1}{2})P_{2n-1}(\frac{1}{2}) + \dots + P_{2n}(-\frac{1}{2})P_0(\frac{1}{2})$$

Draw the graph for Legendre's polynomial P_0, P_1, P_2, P_3, P_4.

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