Write the following information in the first page of Answer Script before starting answer

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number_____

Course_____ Semester_____

Paper Code _____Paper Title _____

Type of Exam: _____ (Regular/Back/Improvement)

Important Instruction for students:

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

M.Sc. MATHEMATICS
THIRD SEMESTER
GENERAL MATHEMATICS-I
MSM-306 (MDC)

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. Find the gcd of 2406 and 654.
a. 7 **b.** 6
c. -7 **d.** None

2. How many ways the letters of the word COMMERCE can be arranged?
a. 7200 **b.** 1540
c. 6040 **d.** 5040

3. If two sets A and B don't have any element in common, then A and B are said to be:
a. Distinct **b.** Disjoint
c. Adjoint **d.** Commutative

4. Find the determinant of $\begin{bmatrix} 5 & 2 & 3 \\ 7 & 3 & 4 \\ 9 & 4 & 5 \end{bmatrix}$
a. 1 **b.** 2
c. 0 **d.** None

5. Find the co-factor of 7 in the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 5 & 1 \\ 3 & 7 & 1 \end{bmatrix}$
a. -1 **b.** 2
c. 1 **d.** None

6. Find two positive numbers such that their AM is 10 and GM is 8.
a. 4 and 14 **b.** 6 and 16
c. 4 and 16 **d.** None

7. If A, B *and* C are three angles of a triangle then $tanA + tanB + tanC = ?$
a. $tanA + tanB\ tanC$ **b.** $tanA\ tanB\ tanC$
c. $tanA\ tanB\ cotC$ **d.** None

8. The no. of elements in a set is called:
a. Subset **b.** Power set
c. Cardinality of the set **d.** None

9. If 4th and 7th term of an HP are $\frac{2}{3}$ and $\frac{2}{5}$ respectively, find the first term.
a. -2 **b.** 3
c. 2 **d.** None

10. What is the value of $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$

a. $\tan\theta$ **b.** $\sin 2\theta$
c. $\tan 2\theta$ **d.** None

11. The unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ is:

a. $\sqrt{14}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$ **b.** $-2\hat{\imath} - 3\hat{\jmath} + \hat{k}$
c. $\frac{1}{\sqrt{14}}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$ **d.** None

12. The function $f(x) = x^2, \forall x \in \mathbb{R}$ is:

a. One-one function **b.** Onto function
c. Both one-one & onto function **d.** None

13. The value of $\lim_{x\to 0}\frac{e^x-1}{x}$ is:

a. 0 **b.** 1
c. -1 **d.** None

14. The direction cosines of $\hat{\imath} + \hat{\jmath} - 2\hat{k}$

a. $(1, 1, -2)$ **b.** $(-1, -1, 2)$
c. $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$ **d.** $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

15. The differential equation $y = x\left(\frac{dy}{dx}\right) + a\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}}$ has:

a. Order 1, Degree 2 **b.** Order 2, Degree 1
c. Order 1, Degree 1 **d.** None

16. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect?

a. $\vec{b} = \lambda\vec{a}$, for some λ **b.** $\vec{b} = \pm\vec{a}$
c. Both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes **d.** The respective components of $\vec{a}$ and $\vec{b}$ are not proportional

17. If $f(x) = b\frac{x-a}{b-a} + a\frac{x-b}{a-b}$, then:

a. $f(a) + f(b) = f(a + b)$ **b.** $f(a) - f(b) = f(a + b)$
c. $f(a) + f(b) = f(a - b)$ **d.** None

18. The lines $2x + 7y - 4 = 0$ and $4x + 14y + 1 = 0$ are:

a. Parallel **b.** Perpendicular
c. Both (a) and (b) **d.** None

19. The centre of the circle $3x^2 + 3y^2 - 6x + 12y - 5 = 0$ is:

a. (1, 2) **b.** (-1, 2)
c. (1, -2) **d.** None

20. The angle between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ is:

a. $\cos^{-1}\left(\frac{1}{3}\right)$ **b.** $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
c. $\cos^{-1}\left(\sqrt{3}\right)$ **d.** None

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PART-B : Descriptive

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. **a**. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $-\hat{\imath}+\hat{\jmath}+\hat{k}$ respectively, in the ratio 2:1 (i) Internally (ii) Externally. 4+3+3=10

b. Find the unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=3\hat{\imath}-7\hat{\jmath}+7\hat{k}$ and $\vec{b}=3\hat{\imath}-2\hat{\jmath}+2\hat{k}$.

c. Prove that – For any two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}|\leq|\vec{a}|+|\vec{b}|$.

2. **a.** Find the perpendicular distance of a point $A(2,3)$ from the line $3x-4y+1=0$. 2+4+4=10

b. Find the angle between the line $2x+y-3=0$ and $x+3y+2=0$.

c. Prove that the four points of intersection of the lines $2x-y+1=0$ & $x-2y+3=0$ with the axes lie on a circle. Find its centre and radius.

3. **a.** If the function $f:\mathbb{R}\to\mathbb{R}$ defined as 3+3+4=10

$$f(x)=\begin{cases}3x-4, & x>0\\ -3x+2, & x\leq 0\end{cases}$$

Determine

(i) $f(0), f\left(\frac{-2}{3}\right)$

(ii) $f^{-1}(0), f^{-1}(2), f^{-1}(-7)$

b. Evaluate $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

c. A function $f(x)$ is defined as follows:

$$f(x)=\begin{cases}-x, & x<0\\ x, & 0\leq x\leq 1\\ 2-x, & x>1\end{cases}$$

(i) Is the function continuous at $x=0$?

(ii) Is the function continuous at $x=1$?

4. **a.** Find the unit vector in the direction of the sum of the vectors $\vec{a}=2\hat{\imath}+2\hat{\jmath}-5\hat{k}$ and $\vec{b}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$. 4+3+3=10

b. Show that the points $A(2\hat{\imath}-\hat{\jmath}+\hat{k}), B(\hat{\imath}-3\hat{\jmath}-5\hat{k}), C(3\hat{\imath}-4\hat{\jmath}-4\hat{k})$ are the vertices of a right angled triangle.

c. If $\vec{a}=5\hat{\imath}-\hat{\jmath}-3\hat{k}$ and $\vec{b}=\hat{\imath}+3\hat{\jmath}-5\hat{k}$, then show that the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

5. **a**. State law of triangle of forces. State and prove Lami's theorem. 1+5+4=10

b. The resultant of two forces $\vec{P}$ and $\vec{Q}$ acting at an angle θ is $\vec{R}$, which makes an angle α with $\vec{P}$. If $P = 4, Q = 5, R = \sqrt{21}$, find θ and α.

6. **a.** There are two groups in a question paper having five questions in each group. An examinee is to answer any six questions. But he cannot answer more than four questions from either of the groups. In how many ways an examinee can select questions? 4+3+3=10

b. **(i)** If the second and fifth terms of an arithmetic progression are 7 and 19 respectively. Find the tenth term and the sum of the first ten terms.

(ii) Find the sum of the series $4 + 44 + 444 + \cdots$ to nth term.

7. 5+5=10

a. Find the inverse of the following matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

b. If a,b,c are in HP, Prove that $\frac{b+a}{b-a} + \frac{b+c}{b-c} = 2$

8. **a.** Solve for x (x is acute and positive) 4+3+3=10

$81^{\sin^2 x} + 81^{\cos^2 x} = 30$

b. **(i)** Show that $cos130^0 + cos110^0 + cos10^0 = 0$

(ii) If $\alpha + \beta = \pi/4$ then prove that $(1 + tan\alpha)(1 + tan\beta) = 2$

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