Chapter-III

Analytical Solution for Steady Magnetohydrodynamic mixed Convection Transport in a Porous Media with Thermal Radiation and Ohmic Heating

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3.1. Introduction:

The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate studied by Soundalgekar *et al.* (1979). MHD effects on impulsively started vertical plate with variable temperature in the presence of transverse magnetic field were considered by Soundalgekar *et al.* (1981). Free convection flows in a porous media with chemical reaction have wide applications in geothermal and oil reservoir engineering as well as in chemical reactors of porous structure. Many transport processes exist in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of diffusion of chemical species. Moreover, considerable interest has been evinced in radiation interaction with convection and chemical reaction for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing–emitting fluids. Khair and Bejan (1985) studied heat and mass on flows past an isothermal flat plate. Lin and Wu (1995) analyzed combined heat and mass transfer by laminar natural convection from a vertical plate. Yin (1999) studied numerically the force convection effect on magnetohydrodynamics heat and mass transfer of a continuously moving permeable surface. Hazarika and Sarma (2011) investigated about the Heat and mass transfer flow along a vertical plate under the combined buoyancy force of thermal and species diffusion in the presence of a transverse magnetic field. Acharya *et al*. (1999) have studied heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing. Muthucumaraswamy and Janakiraman (2006) studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Hossain *et al*. (1999) investigated radiation effects on the free convection flow of an optically incompressible fluid along a uniformly heated vertical infinite plate with a constant suction. Orhan and Kaya (2008) examined MHD mixed convective heat transfer along a permeable vertical infinite plate in the presence of radiation and solutions are derived using Kellar box scheme and accurate finite-difference scheme. Ahmed and

Liu (2010) examined the effects of mass transfer on a mixed convection three dimensional heat transfer flow of a viscous incompressible fluid past an infinite vertical porous plate in the presence of transverse periodic suction velocity. The problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface is analyzed by Chen (2004) by taking into account the effects of Ohmic heating and viscous dissipation but neglected chemical reaction of the species. Ahmed and Hazarika (2012) presented a parametric study to investigate the effects of magnetic field, chemical reaction, thermal radiation, thermal diffusion (Soret effect) and diffusion - thermal (Dufour effect) on a free and forced convective fully developed boundary layer mass transfer flow of an electrically conducting viscous incompressible optically thick fluid past a semi-infinite vertical porous plate. Chaudhury *et al*. (2006) have analyzed the effect of radiation on heat transfer in MHD mixed convection flow with simultaneous thermal and mass diffusion from an infinite vertical plate with viscous dissipation and Ohmic heating. The classical model introduced by Cogley *et al*. (1968) is used for the radiation effect as it has the merit of simplicity and enables us to introduce linear term in temperature in the analysis for optically thin media. The thermal radiation and Darcian drag force MHD unsteady thermal-convection flow past a semi-infinite vertical plate immersed in a semi-infinite saturated porous regime with variable surface temperature in the presence of transversal uniform magnetic field have been discussed by Ahmed *et al*. (2014).

In this problem, it is proposed to study the effects of viscous dissipation and Ohmic dissipation on steady magnetohydrodynamic mixed convection heat and mass transfer flow of a Newtonian, electrically conducting, viscous, incompressible radiative fluid over a porous vertical plate embedded in a porous medium taking into the account of combined effects of buoyancy force and first-order chemical reaction. The governing equations for this investigation are formulated and solved using perturbation technique.

3.2 Mathematical formulation:

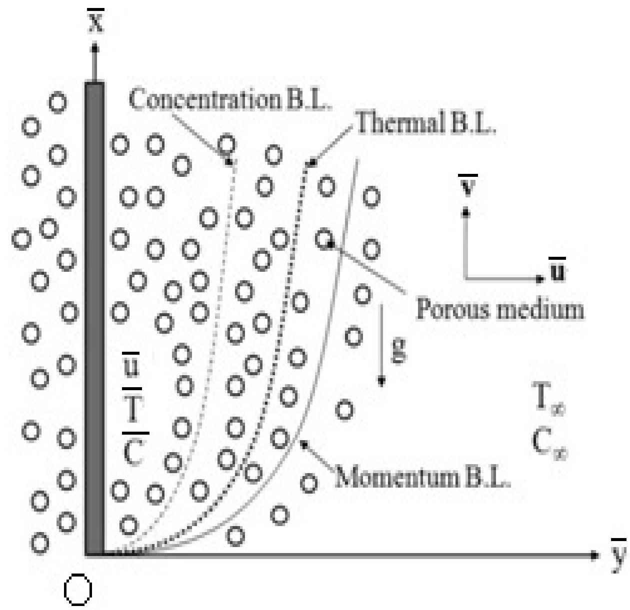


Fig. 3.2 (i): Flow model of the problem

Consider laminar boundary layer flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past a semi-infinite vertical permeable plate embedded in a uniform porous medium which is subject to thermal and concentration buoyancy effects. As shown in Fig. 3.2 (i), $\bar{x}$-axis is along the plate and $\bar{y}$ is perpendicular to the plate. The wall is maintained at a constant temperature T_w and concentration C_w higher than the ambient temperature T_∞ and concentration C_∞, respectively. Also, it is assumed that there exists a homogeneous chemical reaction of first-order with rate constant R between the diffusing species and the fluid. The concentration of the diffusing species is very small in comparison to other chemical species, the concentration of species far from the wall, C_∞, is infinitesimally small and hence the Soret and Dufour effects are neglected. The chemical reactions are taking place in the flow and all thermo-physical properties are assumed to be constant except density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq approximation. It is also assumed that viscous and electrical dissipation are negligible. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of radiation and concentration buoyancy effects in the direction of $\bar{y}$-axis. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall effect are negligible. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. Rest of properties of the fluid and the porous medium are assumed to be constant.

Under these assumptions, the governing equations of the Newtonian flow model of electrically conducting radiative and chemically reacting fluid through porous medium in the presence of magnetic field with heat generation and viscous dissipative heat are (Chaudhary *et al*. (2006)):

$$\frac{d\bar{v}}{d\bar{y}} = 0 \Rightarrow \bar{v} = -v_0 \ (Constant) \tag{3.2.1}$$

$$\frac{d\bar{p}}{d\bar{y}} = 0 \Rightarrow \ \bar{p} \ is \ independent \ of \ \bar{y}$$

$$\rho\bar{v}\frac{d\bar{u}}{d\bar{y}} = \mu\frac{d^2\bar{u}}{d\bar{y}^2} - \frac{\mu}{\bar{K}}\bar{u} - \sigma B_0^2\bar{u} + \rho g\beta_T(\bar{T} - T_\infty) + \rho g\beta_C(\bar{C} - C_\infty)\,, \tag{3.2.2}$$

$$\rho C_P\bar{v}\frac{d\bar{T}}{d\bar{y}} = \alpha\frac{d^2\bar{T}}{d\bar{y}^2} + \mu\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 - \frac{\partial\bar{q}}{\partial\bar{y}} + \sigma B_0^2\bar{u}^2 - Q_0(\bar{T} - T_\infty)\,, \tag{3.2.3}$$

$$\bar{v}\frac{d\bar{C}}{d\bar{y}} = D\frac{d^2\bar{C}}{d\bar{y}^2} - R(\bar{C} - C_\infty)\,, \tag{3.2.4}$$

where v_0 is a scale of suction velocity which has non-zero positive constant, $\bar{u}$ and $\bar{v}$ are the components of dimensional velocities along $\bar{x}$ and $\bar{y}$ directions respectively, α is the fluid thermal diffusivity. The fourth and fifth terms on RHS of the momentum equation (3.2.2) denote the thermal and concentration buoyancy effects, respectively. Also second and fourth terms on the RHS of energy equation (3.2.3) represent the viscous dissipation and Ohmic dissipation, respectively. The third and fifth term on the RHS of Eq. (3.2.3) denote the inclusion of the effect of thermal radiation and heat absorption effects, respectively.

For the radiative heat flux using the Cogley model (1968) is given

$$\frac{\partial \bar{q}}{\partial \bar{y}} = 4(\bar{T} - T_\infty)\bar{I} \tag{3.2.5}$$

$$\bar{I} = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial \bar{T}} d\lambda \tag{3.2.6}$$

$K_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$is Planck's function.

The appropriate boundary conditions for velocity, temperature and concentration fields are

$$\left\{\begin{matrix} \bar{y} = 0: \quad \bar{u} = 0,\ \bar{T} = T_w,\ \bar{C} = C_w, \\ \bar{y} \to \infty: \quad \bar{u} \to 0,\ \bar{T} \to T_\infty,\ \bar{C} \to C_\infty, \end{matrix}\right\} \tag{3.2.7}$$

where C_w and T_w are the wall dimensional concentration and temperature respectively.

Introducing the following non-dimensional quantities:

$$\left\{\begin{matrix} y = \dfrac{v_0 \bar{y}}{\nu},\ u = \dfrac{\bar{u}}{v_0},\ M^2 = \dfrac{\sigma B_0^2 \nu^2}{\mu v_0^2},\ K = \dfrac{\bar{K} v_0^2}{\nu^2}, \\ \theta = \dfrac{\bar{T} - T_\infty}{T_w - T_\infty},\ \phi = \dfrac{\bar{C} - C_\infty}{C_w - C_\infty}, \end{matrix}\right\} \tag{3.2.8}$$

Using (3.2.5) and (3.2.8) in equations (3.2.2)–(3.2.4), we get the following non-dimensional equations:

$$\frac{d^2u}{dy^2} + \frac{du}{dy} - (M^2 + K^{-1})u = Gr\,\theta - Gm\phi\ , \tag{3.2.9}$$

$$\frac{d^2\theta}{dy^2} + Pr\frac{d\theta}{dy} + Pr\,Ec\left(\frac{du}{dy}\right)^2 - Pr(Ec + \psi)\theta + PrEcM^2u^2 = 0\ , \tag{3.2.10}$$

$$\frac{d^2\phi}{dy^2} + Sc\frac{d\phi}{dy} - Sc\gamma\phi = 0\,, \tag{3.2.11}$$

where Gr is the Grashoff number, Gm is the solutal Grashoff number, Pr is the Prandtl number, M is the magnetic field parameter, F is the radiation parameter, Sc is the Schmidt number, Ec is the Eckert number, ψ is the heat source parameter and γ is the chemical reaction parameter and K is the permeability parameter defined as follows:

$$\left\{\begin{array}{c} Ec = \dfrac{v_0^2}{C_P(T_w - T_\infty)} \ , \ \ Pr = \dfrac{\mu C_P}{\alpha} \ \ , \ \ Gr = \dfrac{\rho g \beta_T (T_w - T_\infty)}{\mu v_0^3} \\ Gm = \dfrac{\rho g \beta_C (C_w - C_\infty)}{\mu v_0^3} \ \ , \ \ Sc = \dfrac{\nu}{D} \ , \ \ \gamma = \dfrac{R\nu}{v_0^2} \ \ , \\ \psi = \dfrac{Q_0 \nu}{\rho C_P v_0^2} \ , \ \ F = \dfrac{4\nu I'}{\rho C_P v_0^2} \end{array}\right\} \tag{3.2.12}$$

The corresponding boundary conditions in dimensionless form are

$$\left\{\begin{array}{ll} y = 0: & u = 0, \ \ \theta = 1, \ \ \phi = 1 \\ y \to \infty: & u \to 0, \ \ \theta \to 0, \ \ \phi \to 0 \end{array}\right\} \tag{3.2.13}$$

3.3 Method of solution:

The physical variables the velocity u, temperature θ and concentration ϕ can be expanded in terms of power of Eckert number Ec. This can be possible physically as in the flow of an incompressible fluid Eckert number is always less than unity since the flow due to the Joules dissipation is super imposed on the main flow. Hence, we can assume

$$\left\{\begin{array}{l} u(y) = u_0(y) + Ecu_1(y) + o(Ec^2) \\ \theta(y) = \theta_0(y) + Ec\theta_1(y) + o(Ec^2) \\ \phi(y) = \phi_0(y) + Ec\phi_1(y) + o(Ec^2) \end{array}\right\} \tag{3.3.1}$$

Substituting (3.3.1) in equations (3.2.9)–(3.2.11) and equating the coefficient of zeroth powers of Ec $(i.e. O(Ec^0))$, we get the following set of equations:

$$u_0'' + u_0' - \mathrm{N}u_0 = -Gr\theta_0 - Gm\phi_0 \tag{3.3.2}$$

$$\theta_0'' + Pr\theta_0' - Pr(F + \psi)\,\theta_0 = 0 \tag{3.3.3}$$

$$\phi_0'' + Sc\phi_0' - \mathrm{Sc}\gamma\phi_0 = 0 \tag{3.3.4}$$

Next, equating the coefficients of first-order of Ec $(i.e. O(Ec^1))$, we obtain

$$u_1'' + u_1' - \mathrm{N}u_1 = -Gr\theta_1 - Gm\phi_1\,, \tag{3.3.5}$$

$$\theta_1'' + Pr\theta_1' - Pr \quad (F+\psi)\,\theta_1 + Pr{u_0^2}' + PrM^2u_0^2 = 0\,, \tag{3.3.6}$$

$$\phi_1'' + Sc\phi_1' - Sc\gamma\phi_1 = 0\,, \tag{3.3.7}$$

where $N = M^2 + K^{-1}$.

The corresponding boundary conditions are

$$\left\{\begin{array}{l} y=0: \quad u_0=0,\ \ u_1=0,\ \ \theta_0=1,\ \ \theta_1=0\,,\ \ \phi_0=1\,,\ \ \ \phi_0=0 \\ y\to\infty: \quad u_0\to 0,\ \ u_1\to 0,\ \ \theta_0\to 0,\ \ \theta_1\to 0,\ \ \phi_0\to 0\,,\ \ \phi_1\to 0 \end{array}\right\} \tag{3.3.8}$$

We have restricted the solution of velocity, temperature and concentration fields up to $O(Ec)$ and neglected the higher order of $O(Ec^2)$ as the value of $Ec \ll 1$. Without going into the details, solutions of equations (3.3.2)–(3.3.7) with the help of boundary conditions (3.3.8) and (3.3.9) are obtained as follows:

$$u_0 = A_5(e^{-A_4y} - e^{-A_1y}) + A_6(e^{-A_4y} - e^{-m_1y}), \tag{3.3.10}$$

$$\theta_0 = e^{-A_1y}\,, \tag{3.3.11}$$

$$\phi_0 = e^{-m_1y}\,, \tag{3.3.12}$$

$$u_1 = \left\{\begin{array}{c} A_{17}e^{-A_4y} - B_{10}e^{-A_1y} + B_{11}e^{-2A_1y} + B_{12}e^{-2A_4y} \\ -B_{13}e^{-A_{10}y} + B_{14}e^{-2m_1y} - B_{15}e^{-B_1y} + B_{16}e^{-B_2y} \end{array}\right\}, \tag{3.3.13}$$

$$\theta_1 = \left\{\begin{array}{c} B_9e^{-A_1y} - B_3e^{-2A_1y} - B_4e^{-2A_4y} + B_5e^{-A_{10}y} \\ -B_6e^{-2m_1y} + B_7e^{-B_1y} - B_8e^{-B_2y} \end{array}\right\}\,, \tag{3.3.14}$$

$$\phi_1 = 0 \tag{3.3.15}$$

The physical quantities of interest are the wall shear stress τ_w is given by

$$\tau_w = \mu \frac{\partial \bar{u}}{\partial \bar{y}}\bigg|_{\bar{y}=0} = \rho v_0^2 u'(0)$$

$$C_{f_x} = \frac{\tau_w}{\rho v_0^2} = u'(0) \tag{3.3.16}$$

Using (3.3.1), (3.3.10) and (3.3.13) in (3.3.16), we get

$$C_{f_x} = \begin{Bmatrix} A_6(m_1 - A_4) + A_5(A_1 - A_4) - Ec(B_{17}A_4 - B_{10}A_1 + 2B_{11}A_1 \\ +2B_{12}A_4 - B_{13}A_{10} + 2B_{14}m_1 - B_{15}B_1 + B_{16}B_2) \end{Bmatrix}.$$

The local surface heat flux is given by

$$q_w = -\kappa \frac{\partial \bar{T}}{\partial \bar{y}}\bigg|_{\bar{y}=0},$$

where κ is the effective thermal conductivity.

The local Nusselt number

$Nu_x = q_w/(T_w - T_\infty)$ can be written as

$$\frac{Nu_x}{Re_x} = A_1 Ec[B_9A_1 - 2B_3A_1 - 2B_4A_4 + B_5A_{10} - 2B_6m_1 + B_7B_1 - B_2B_8],$$

where $Re_x = v_0 X/\nu$ is the local Reynolds number.

3.4 Validity:

Validity of the analysis has been performed by comparing the present results with those available in the open literature i.e. Chaudhary *et al*. (2006) and a very good agreement has been established, when K=∞, ψ=0.0, γ=0.0. In order to verify the accuracy of the present results, we have considered the analytical solutions obtained by Chaudhary *et al*. (2006) and computed these solutions for various physical parameters for skin-friction coefficient and local Nusselt number.

Table- 3.4 (*a*):

Comparison of present results with those of Chaudhary *et al*. (2006) with different values of F for C_{fx} ; with Pr = 0.71, Sc = 0.78, M = 5.0, Gr = 5.0, Gm = 5.0, Ec = 0.05.

Chaudhary *et al*. (2006)			Present results		
F	C_{fx}	Nu_x/Re_x	F	C_{fx}	Nu_x/Re_x
1.0	1.82701	1.52710	1.0	1.82981	1.52803
2.0	1.77315	1.82047	2.0	1.77918	1.82209
3.0	1.73182	2.17602	3.0	1.73423	2.17803
4.0	1.60718	2.32635	4.0	1.60817	2.32725
5.0	1.52541	2.50981	5.0	1.52598	2.51083

The comparison table shows that the adopted flow model is validated in comparison with Chaudhary *et al*. (2006) as the absolute difference between the present results and Chaudhary *et al*. (2006) is very less than to unity ($<<10^{-3}$). Moreover, it is observed that the local skin friction is reduced when the thermal radiation increased, but a reversed effect has occurred for the local Nusselt number and the heat is able to diffuse from surface to the fluid region.

3.5 Results and Discussion:

To get a physical insight into the problem the numerical evaluation of the analytical results reported in the previous section has been performed and a set of results reported graphically in Figures 3.5 (i)-3.5 (vi) for the cases cooling $Gr>0$ of the plate i.e. free convection currents convey heat away from the plate into the boundary layer. During the numerical calculations the physical parameters are considered as $Pr = 0.71$ (diffusing air), $Gr = 5$ (thermal buoyancy forces are dominant over the viscous hydrodynamic forces in the boundary layer), $F = 5>1$ (thermal radiation is dominant over the thermal conduction), $Ec = 0.05\leq 1$ (Enthalpy difference is dominant over the kinetic energy).

Figs. 3.5 (i) and 3.5 (ii) illustrate the influence of the heat absorption and porosity parameters ψ and K, respectively on the flow velocity. The effect is observed on velocity profile by increasing the value of the heat absorption parameter ψ and the boundary layer thickness decreases with increase in the absorption parameter as shown in Fig. 3.5 (i), which is expected. The opposite trend is observed in Fig. 3.5 (ii) for the case when the value of the porous permeability is increased. As depicted in this figure, the effect of increasing the value of porous permeability is to increase the value of the velocity component in the boundary layer due to the fact that drag is reduced by increasing the value of the porous permeability on the fluid flow which results in increased velocity. Fig. 3.5 (iii) depicts the effect of radiation on the flow velocity. We note from this figure that there is decrease in the value of flow velocity with increase in radiation parameter F which shows the fact that increase in radiation parameter decrease the velocity in the boundary layer due to decrease in the boundary layer thickness. The effect of chemical reaction parameter γ is highlighted in Fig. 3.5 (iv) which shows that the velocity is decreased with increasing the rate of chemical reaction γ. Hence increase in the chemical reaction rate parameter leads to a fall in the momentum boundary layer. The trend of the velocity profile in this figure is same as shown in Fig. 3.5 (iii). The effect of absorption parameter (ψ) on fluid temperature (θ) is presented in Fig. 3.5(v). This illustrates that the escalating values of absorption parameter (ψ) declines the temperature profiles and it is due to the fact that the thermal boundary layer absorbs energy which causes the temperature fall considerably with increasing the value of internal heat absorption parameter. The effects of the reaction rate parameter (γ) on the species concentration profiles ($\emptyset$) for generative chemical reaction are shown in Fig. 3.5(vi). It is noticed from the graphs that there is a decreasing effect on concentration distribution with the increasing value of the chemical reaction rate parameter in the boundary layer.

Table 3.4 (a) shows that the values of skin friction coefficient are decreased and that of local Nusselt number increased with the increasing value of radiation parameter F.

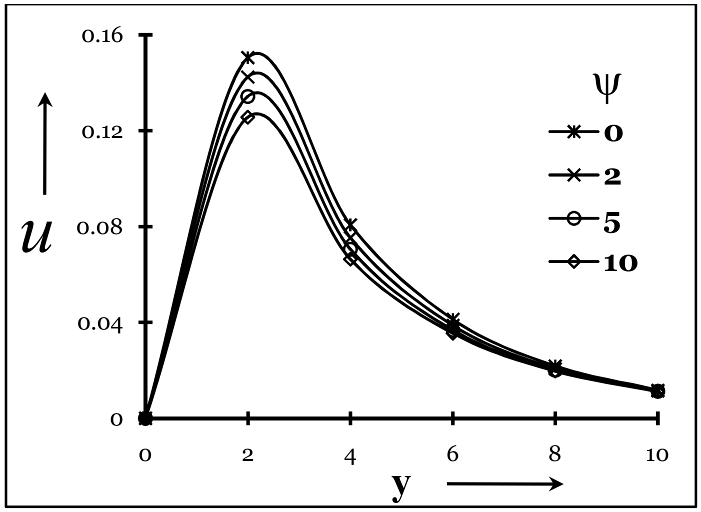
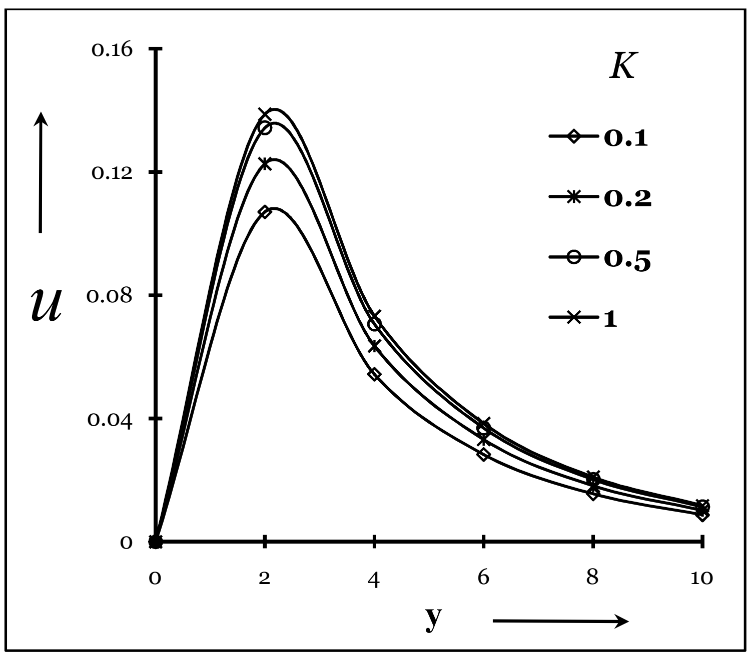


Fig. 3.5 (i): Velocity distribution for heat absorption (ψ)



3.5 (ii): Velocity distribution for porosity (*K*)

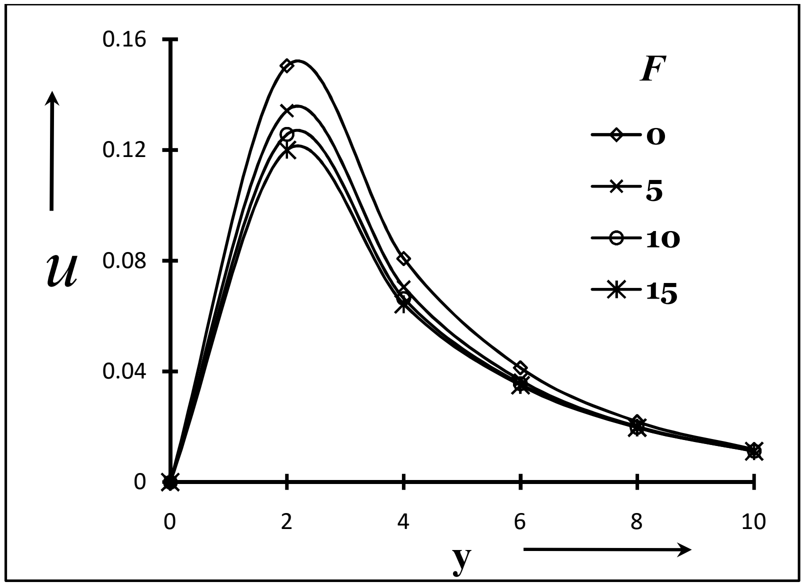


Fig. 3.5 (iii): Velocity distribution for radiation (*R*)

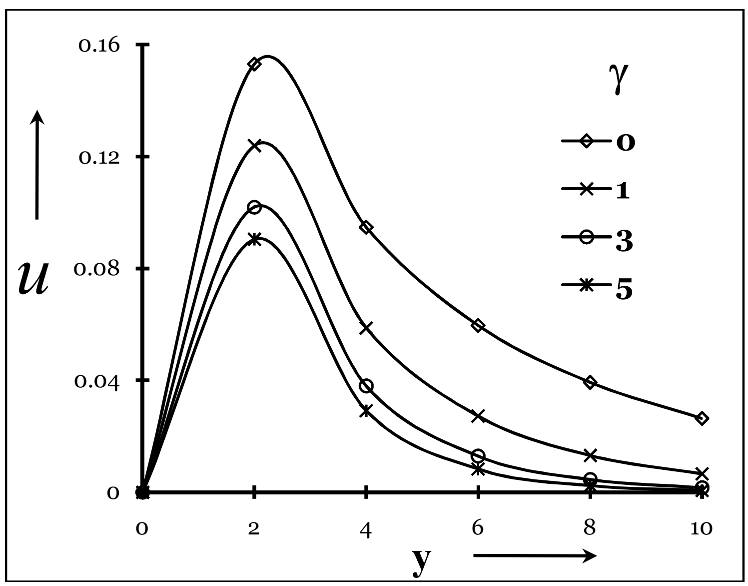


Fig. 3.5(iv): Velocity distribution for chemical reaction (γ)

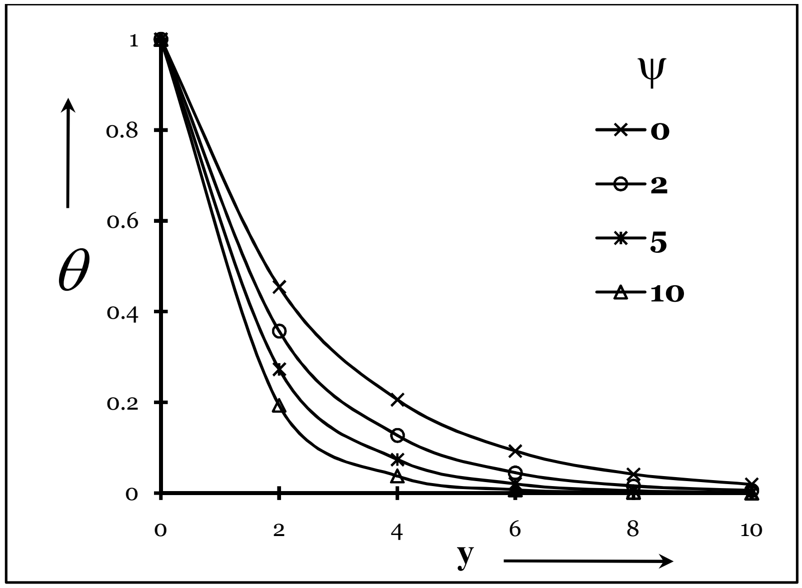


Fig. 3.5 (v): Temperature for heat absorption (ψ)

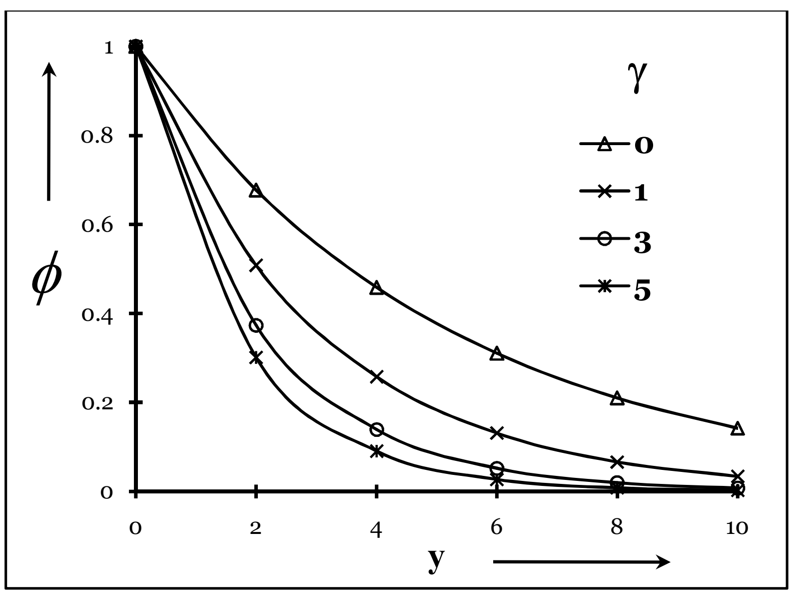


Fig. 3.5(vi): Temperature for chemical reaction (γ)

3.6 Conclusions:

A theoretical analysis of the steady magnetohydrodynamic flow and mixed convection heat and mass transfer in a viscous, incompressible, electrically-conducting fluid along a semi-infinite vertical plate immersed in a porous medium with thermal radiation has been conducted. The flow model has been setup for homogeneous chemical reaction of first-order in the presence of Ohmic heating and viscous dissipation. The nonlinear and coupled governing equations are solved analytically by perturbation technique. The computations have shown that:

- It is seen that the velocity starts from minimum value of zero at the surface and increases till it attains the peak value and then starts decreasing until it reaches the minimum value at the end of the boundary layer.
- Increasing *heat absorption* acts to decelerate the flow velocity in the boundary layer
- Flow velocity is accelerated with increasing porosity parameter in the porous regime.
- It is seen that with an increase in *heat absorption* of the steady motion, the temperature is decreased.
- For the steady state case, there is a strong reduction in the concentration distribution for the effect of *generative chemical reaction*.