CHAPTER- II

Magnetohydrodynamic Oscillatory Flow in a Planer Porous Channel with Suction and Injection

Some part of this chapter has been published in the UGC Approved Journal entitled- "International Journal of Engineering Research & Technology" (IJERT), ISSN: 2278-0181, Vol. 2 Issue 11, November – 2013.

2.1. Introduction:

Magnetohydrodynamics deals with dynamics of an electrically conducting fluid, which interacts with a magnetic field. The study of MHD flow, through and across porous media, is of great theoretical interest because it has been applied to a variety of geophysical and astrophysical phenomena. Practical interest of such study includes applications in electromagnetic lubrication, boundary cooling, bio-physical systems and in many branches of engineering and science. In fact, flows of fluids through porous media have attracted the attention of a number of scholars because of their possible applications in many branches of science and technology. In fact a porous material containing the fluid is a non-homogeneous medium but it may be possible to treat it as a homogeneous one, for the sake of analysis, by taking its dynamical properties to be equal to the averages of the original non-homogeneous continuum. Thus a complicated problem of the flow through a porous medium gets reduced to the flow problem of a homogeneous fluid with some additional resistance. The hydrodynamic channel flow is a classical problem for which exact solution can be obtained Schllicting (1979). Eckert (1958) obtained the exact solution of Navier-Stokes equations for the flow between two parallel porous plates with constant injection/suction.

In view of numerous important engineering and geophysical applications of the channel flows through porous medium, for example in the fields of chemical engineering for filtration and purification processes, in the fields of agriculture, engineering for channel irrigation and to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the oil channel/reservoirs. A series of investigations have been made by different scholars like Ahmadi and Manvi (1971); Raptis (1983); Raptis and Perdikis (1985); Singh and Garg (2010) and Singh and Sharma (2001) where the porous medium is either bounded by a channel or by a plane surface. On the other hand, in view of the increasing technical applications using magnetohydrodynamic (MHD) effect, it is desirable to extend many of the available hydrodynamic solution to include the effects of magnetic field for those cases where the viscous fluid is electrically conducting. The effect of a transverse magnetic field on free convective flows of an electrically conducting viscous fluid has been discussed in recent and past years by several authors, notably by Gupta 1969), Soundalgekar (1974), Mishra and Mudili (1976), Mahendra (1977), Sarojamma and Krishna (1981) and Singh and Garg (2009). Such types of flows have wide range of applications in aeronautics, fluid fuel nuclear reactors and chemical engineering. The various applications of MHD flows in technological fields have been complied by Moreau (1990). Recently Makinde and Mhone (2005) investigated the effects of radiative heat and magnetic field on the unsteady flow of a fluid through a channel filled with saturated porous media. This problem is further extended by Mehmood and Ali (2007) by considering the fluid slip conditions at the stationary plate. Major mistakes found in both the above (2005), (2007) studies had been marked by Singh and Garg (2010). Kuznetsov (1998) presented an analytical solution to the flow and heat transfer in Couette flow through a rigid saturated porous medium where the fluid flow occurs due to a moving wall and it is described by the Brinkman-Forchheimer-extended Darcy equation. The problem of free convection heat transfer flow through a porous medium bounded by a wavy wall and a vertical wall is studied by Ahmed (2008). A three-dimensional Couette flow through a porous medium with heat transfer has also been investigated by Ahmed (2009). Ahmed and Zueco (2011) investigated the effects of Hall current, magnetic field, rotation of the channel and suction/injection on the oscillatory free convective MHD flow in a rotating vertical porous channel when the entire system rotates about an axis normal to the channel plates and a strong magnetic field of uniform strength is applied along the axis of rotation. Ahmed (2010) investigated the effect of periodic heat transfer on unsteady MHD mixed convection flow past a vertical porous flat plate with constant suction and heat sink when the free stream velocity oscillates in about a non-zero constant mean. Moreover, Ahmed and Kalita (2013) investigated the effects of thermal radiation and magnetohydrodynamic forces on transient flow over a hot vertical plate in a Darcian regime.

The aim of the present paper is to study the combined effects of injection/suction and magnetic field on the oscillatory flow through saturated porous medium bounded by two parallel porous plates.

2.2. Mathematical Analysis:

Consider a two-dimensional flow of a viscous, incompressible, electrically conducting, Newtonian fluid through saturated porous medium filled in an infinite horizontal channel. The plates of the channel are distance 'a' apart. A coordinate system is chosen with $\bar{x}$ –axis lies along the centerline of the channel and $\bar{y}$ - axis is normal to the planes of the plates. Both the lower and the upper stationary porous plates of the channel are subjected to the same constant injection and suction velocity. A homogeneous magnetic field B_0 is applied normal to the planes of the plates as shown in Figure-2.2 (i). The flow becomes oscillatory due to the time dependence of the pressure. All the physical quantities are independent of $\bar{x}$ for the problem of fully developed laminar flow. Under all these assumptions the flow is depicted mathematically as:

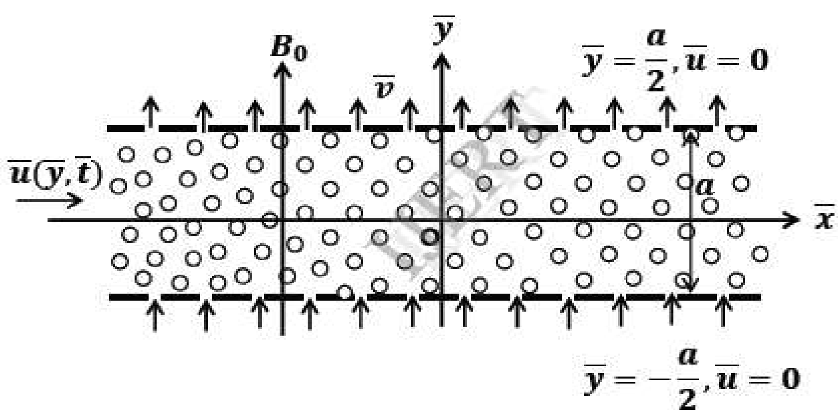

Fig.2.2 (i) Physical Model of the problem

Conservation of mass

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2.2.1}$$

Conservation of momentum

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\,\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho}\bar{u} - \frac{\nu}{\bar{K}} \tag{2.2.2}$$

The boundary conditions of the problem are

$$\left\{\begin{matrix} \bar{u} = 0\,, \bar{v} = V \ \ \text{at } \bar{y} = \dfrac{a}{2} \\ \bar{u} = 0\,, \bar{v} = V \ \ \text{at } \bar{y} = -\dfrac{a}{2} \end{matrix}\right\} \tag{2.2.3}$$

On introducing the following non-dimensional quantities,

$$\left\{\begin{matrix} x = \dfrac{\bar{x}}{a}\,, y = \dfrac{\bar{y}}{a}, u = \dfrac{\bar{u}}{a}\,, t = \dfrac{\bar{t}\nu}{a}\,, p = \dfrac{\bar{p}}{\rho V^2}\,, \\ \lambda = \dfrac{Va}{\nu}\,, Da = \dfrac{\bar{K}V}{\nu a}, M = aB_0\sqrt{\dfrac{\sigma}{\mu}} \end{matrix}\right\} \tag{2.2.4}$$

into the equations (2.2.1) & (2.2.2) we get,

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\lambda}\frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{\lambda}M^2 + \frac{1}{Da}\right)u\,, \tag{2.2.5}$$

where ρ is the density, $\bar{t}$ is time, $\bar{u}$ is the axial velocity, $\bar{v}$ is the transverse velocity, ν is the kinematic viscosity, $\bar{p}$ is the pressure, $\bar{K}$ is constant of permeability of the porous medium, B_0 uniform magnetic field, σ is electrical conductivity, λ is the injection/suction parameter, Da is the Darcy number and *M* is the Hartmann number.

The transformed boundary conditions become

$$\begin{Bmatrix} u = 0\,, v = V \;\; at \;\; y = \dfrac{1}{2} \\ u = 0\,, v = V \;\; at \;\; y = -\dfrac{1}{2} \end{Bmatrix} \tag{2.2.6}$$

2.3 Method of solution:

In order to solve equation (2.2.5) under the boundary conditions (2.2.6), let us assume the solution of the following form

$$u(y,t) = u_0(y)e^{i\omega t}, \qquad -\frac{\partial p}{\partial x} = \Omega e^{i\omega t} \;, \tag{2.3.1}$$

where Ω is constant and ω is the frequency of oscillations.

Substituting expressions (2.3.1) into equations (2.2.5), we obtain

$$u_0'' - \lambda u_0' - m^2 u_0 = -\lambda\Omega \;, \tag{2.3.2}$$

where $m = \sqrt{M^2 + \frac{\lambda}{Da} + i\omega\lambda}$.

The corresponding transformed boundary conditions become

$$\begin{Bmatrix} u = 0\,, \;\text{at}\;\; y = \dfrac{1}{2} \\ u = 0\,, \;\text{at}\;\; y = -\dfrac{1}{2} \end{Bmatrix} \tag{2.3.3}$$

Equation (2.3.2) is solved under boundary conditions (2.3.3) and the solution for the fluid velocity is obtained as under:

$$u(y,t) = \frac{\lambda\Omega}{m^2}\left[1 - \frac{Cosh(A_1 y)}{Cosh\left(\frac{A_1}{2}\right)}\right] e^{i\omega t} \;, \tag{2.3.4}$$

where $A_1 = \frac{1}{2}\left[\frac{\lambda + \sqrt{\lambda^2 + 4m^2}}{2}\right]$.

Knowing the fluid velocity (Real part), the shear stress (Real part) at the lower plate $y = -1/2$ is given by:

$$\tau = -\mu \left(\frac{\partial u}{\partial y}\right)_{y=-1/2} = \frac{\Omega\lambda}{m^2} tanh\left(\frac{A_1}{2}\right) \quad (2.3.5)$$

2.4 Results and Discussion:

The analytical solutions obtained in equation (2.3.4) and (2.3.5) for the velocity and skin friction have been calculated numerically to have a physical insight into the problem. The effects of variations of different parameters like the Darcy number (*Da*), injection / suction parameter (λ), Hartmann number (*M*) and frequency of oscillations (ω) on the velocity field (Real part of u) and skin friction (Real part of τ) variations are presented graphically in Figures 2.4 (i) to 2.4 (iv) below.

Figure 2.4 (i) shows the collective effects of Hartmann number (*M*) and Darcy number (*Da*) on the flow velocity (*u*) for different values of ω = 10, λ = 0.2, Ω= 10. The influence of *M* and *Da* on *u* profiles is therefore expected to be strong. This is indeed the case as seen in fig. 2.4 (i); for constant *Da* (= 0.3), with a rise in *M*, from 1, 5 to 10 there is a strong reduction in velocity across the region *y* □ [−0.5, 0.5]. The flow is therefore decelerated with increasing Hartmann number owing to the corresponding increase in the ***Lorentizian hydromagnetic*** drag force. Moreover, with constant *M* = 1.0 value, as Da increases from 0.3 through 0.5 to 1.0, there is a distinct escalation in velocity across the region *y* □ [−0.5, 0.5]. A velocity peak arises in the middle of the channel for all profiles. No back flow is sustained throughout the channel.

Figure 2.4 (ii) presents the flow velocity profiles for the effect of injection/suction parameter (λ) and frequency of oscillations ω for different values of *M* = 5, *Da* = 0.5, Ω = 10. With constant ω = 5, an increase in λ from 0.1 through 0.5 to 1.0, the flow velocity is accelerated throughout the channel and attains its maximum velocity in the middle of the channel. Moreover with constant λ = 0.1, the flow velocity is decelerated throughout the channel when the frequency of oscillations rises from 5.0 through 10 to 15. No back flow is sustained throughout the channel.

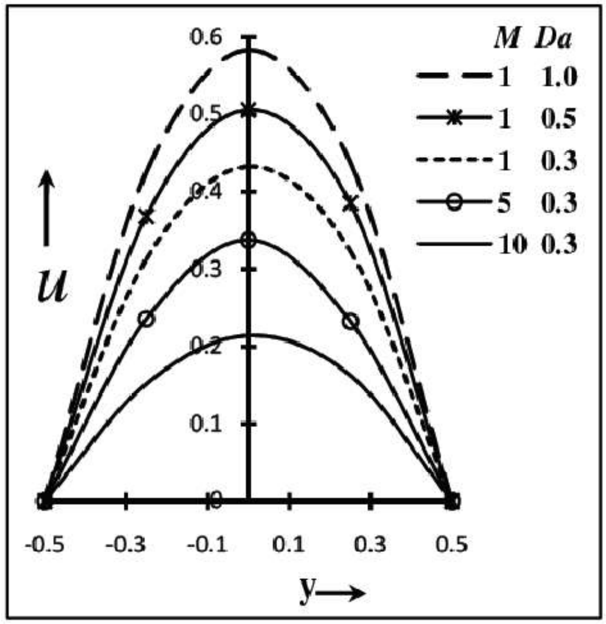

Fig 2.4 (i): Velocity distributions for *M* and *Da* at *t* = 0

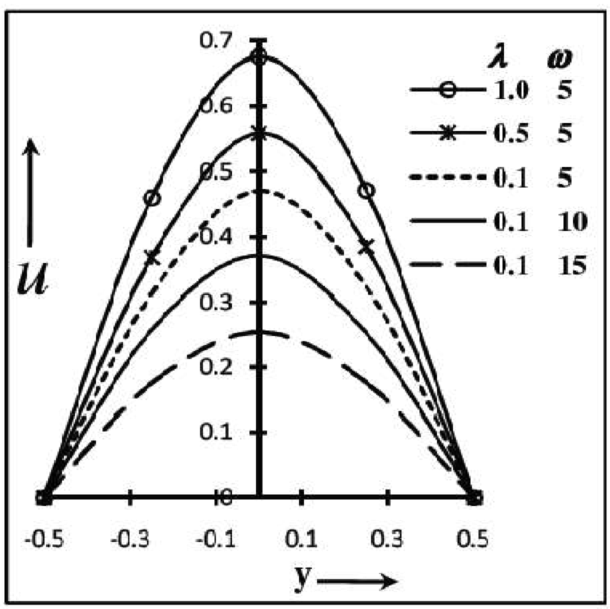

Fig 2.4 (ii): Velocity distributions for λ and *ω* at *t* = 0

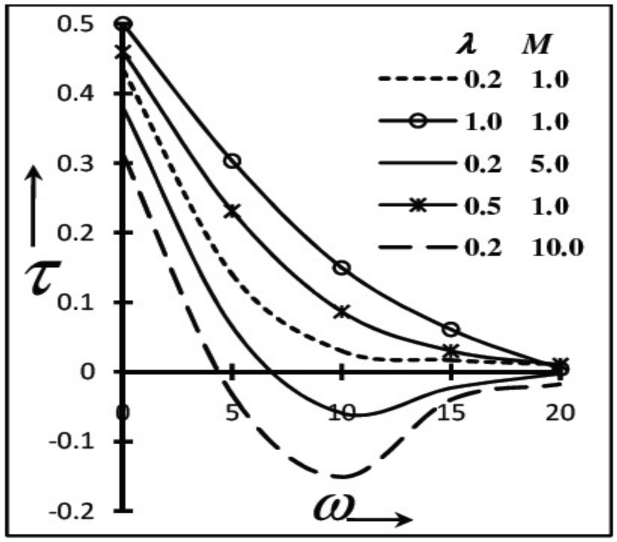

Fig. 2.4 (iii): Variations of Skin friction for M and λ versus *ω*

Figure 2.4(iii) depicts the influence of the applied magnetic field (M) and injection-suction parameter (λ) on the non-dimensional coefficient of skin friction τ at the lower plate for different values of Da = 0.5, Ω = 10. The imposition of the magnetic field causes to decrease τ. It is noticed that, the influence of injection/suction on τ is significantly elevated. The shear stress at the wall is therefore depressed with a rise in Hartmann number (M). For the higher values of ω (> 4.8), a significant flow reversal is sustained with maximum magnetohydrodynamic forces M = 5 and 10 i.e. shear stresses become negative. No flow reversal however arises for small frequency of oscillations. The back flow effect is still present for M = 5 (magnetic body force is five times the viscous hydrodynamic force), but is stifled somewhat and the inception of backflow is further delayed. However for M =1 and λ = 0.2, 0.5 and 1.0, all backflow is eliminated entirely from the regime for all frequency of oscillations and only positive shear stresses arise at the plate. Generally with frequency of oscillations, shear stresses are found to reduce i.e. the flow is retarded.

Figure 2.4(iv) shows the distribution of shear stress at the lower plate for various Darcy numbers over frequency of oscillations for different values of M = 3, λ = 0.5, Ω = 10, Ω = 10. Again it is seen that the shear stresses are reduced substantially at the lower plate throughout the channel. For all $\omega > 4.8$, flow reversal is observed for small Darcy numbers (Da = 0.1 and 0.5) and therefore, back flow is sustained throughout the regime. Shear stresses are significantly boosted with rising Darcy number from 0.1 through 0.5, 0.7 to 1.0. Significantly, shear stress at Da = 1.0 is more fluctuated than the others.

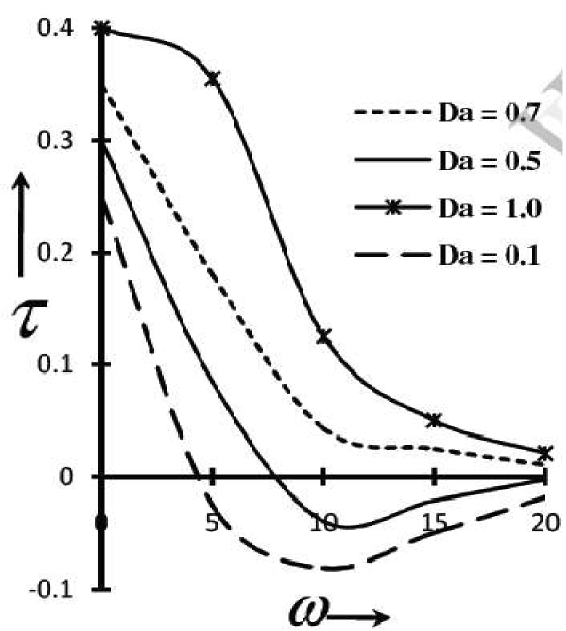

Fig 2.4(iv): Variations of Skin friction for *Da* versus *ω*

2.5 Conclusions:

A theoretical model has been presented for the hydromagnetic unsteady boundary layer flow from a horizontal channel bounded by two parallel plates filled with saturated porous medium with transverse magnetic field effects, subject to a constant suction/injection velocity. Analytical solution for the non-dimensional momentum equation subject to transformed boundary conditions has been obtained. The flow has been shown to be accelerated with increasing suction/injection parameter, but reduced with Magnetic field. Increasing Hartmann number also decreases the shear stresses. A positive increase in Da strongly accelerates the flow. The study has important applications in materials processing and nuclear heat transfer control, as well as MHD energy generators.