Chapter-I

Introduction

1.1 Background of Fluid Mechanics:

The science of fluid Mechanics began to develop into two divergent branches which had practically no points in common and they were the *Theoretical Hydrodynamics* and the other was *Science of Hydraulics*. The theoretical hydrodynamics developed from solutions of Euler's equation of motion for various flow configurations of frictionless or non-viscous fluid past obstacles like cylinders, spheres, through pipes, channels and against disk. However, the results of such studies did not agree with the experimental results as regards to the pressure losses in pipes and channels, as well as with regard to the drag of the body which moves through a mass of fluid. The most glaring departure of the result of this subject from reality being that leading to d' Alembert's paradox that is, to the statement that a body which moves uniformly through a fluid which extends to infinity experiences no drag whereas a body experiences moving through any real fluid. On the other hand, the science of hydraulics was mainly developed by the practical engineers for the need to solve the important problems arising from progress in technology. Here the equations were generally empirical, without much theoretical content. The science of hydraulics was based on a large number of experimental data and differed greatly in its method and in its objects from the theoretical hydrodynamics.

Towards the end of nineteenth century, the equations of motion of a viscous fluid were established by Navier (1823), Poisson (1831), Saint-Venant (1843) and Stokes (1845) and these are known as Navier-Stokes equations. But these equations being non-linear partial differential equations for the cases where either the non-linear terms vanish identically or the equation of motion can be reduced to ordinary differential equations by taking recourse to Laplace transformation or to some suitable similarity transformations. Stokes (1851) investigated the case of parallel flow past a sphere for the limiting case when the viscous forces are considerably greater than the inertia forces and so the non-linear terms in the Navier-Stokes equations are neglected. Oscen (1910) gave an improvement on the Stokes solution by taking partially into account the inertia terms in the Navier-Stokes equation. However these types of solution are valid for small Reynolds number $= \frac{UL}{\nu}$, where U and L are some characteristic velocity and length respectively of the problem and ν is the kinematic viscosity. When L and ν are fixed, low R corresponds to slow motion and do not occur often in practical applications. As a result, there was not much progress till the beginning of the twentieth century in dealing with the flow problems of real fluids by considering the full Navier-Stokes equation along with the no slip condition at a solid wall.

Moreover, the problem of the complete investigation of the fluid flow characteristics is to study the velocity distribution and the state of fluid even the entire space for all time. There are six characteristics to study viz. the three components of velocity (u, v, w), the temperature distribution θ, the pressure p at a point and density ρ of the fluid. The following equations connecting these characteristics form the basis of the problem:

- Equation of the state, which connects temperature, pressure and density.
- Equation of continuity, which formulates mathematically the law of conservation of mass of the fluid.
- Equation of motion, which represent the conservation of momentum of the fluid.
- Equation of energy which gives the relation of the conservation of the energy of the fluid.

Types of fluid:

The fluids may be classified into the following categories:

Ideal fluid

A fluid which is incompressible and having no viscosity, is known as an ideal fluid. Ideal fluid is an imaginary fluid as all the fluids are having some viscosity

Real fluid

A fluid which possesses viscosity is known as real fluid. All the fluids in actual practice are real fluids.

Newtonian fluid

A real fluid in which the shear stress is directly proportional to the rate of shear strain or velocity gradient known as Newtonian fluid.

Non-Newtonian fluid

A real fluid in which the shear stress is not proportional to the rate of shear strain or velocity gradient known as a Non-Newtonian fluid.

Ideal plastic fluid

A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain or velocity gradient, is known as Ideal plastic fluid.

Classification of fluid flows:

There is a wide variety of fluid flow problems encountered in practice and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify fluid flow problems and here we present some general categories.

Steady versus unsteady flows

The term steady implies no change at a point with time. It means that in steady flow the fluid characteristics like velocity, pressure, density etc. at a point do not change with time. Mathematically, we have,

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} = 0 \quad , \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} = 0\,, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0 \;\; etc.$$

where (x_0, y_0, z_0) , is a fixed point in fluid field.

On the contrary the term unsteady is used to any flow that is not steady. It is the flow, in which the velocity, pressure or density at a point change with respect to time. Thus mathematically,

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} \neq 0\,, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} \neq 0\,, \quad etc.$$

Compressible versus incompressible flows

A flow is classified as being compressible or incompressible, depending on the level of variation of density during flow. So in case of compressible flow the density of the fluid changes from point to point but incompressibility is an approximation and the flow is said to be incompressible if the density remains nearly constant throughout. Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow is incompressible.

Laminar versus turbulent flows

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth layers of fluid is called laminar. The word laminar comes from the movement of adjacent fluid particles together in “Laminates”. The flow of high –viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called turbulent. The flow of low-viscosity fluids such as air at high velocities is typically turbulent.

If the Reynolds number is less than 2000, the flow is called laminar and if it is more than 4000 then the flow becomes turbulent. If the number lies between 2000 and 4000, the flow may be laminar or turbulent.

Natural versus forced flows

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In forced flow, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural flows, any fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid.

Uniform and non-uniform flows

The term uniform implies no change with location over a specified region. Thus in uniform flow the velocity at any given time does not change with respect to space (i.e. length of direction of the flow). Mathematically, $\left(\frac{\partial V}{\partial S}\right)_{t=constant} = 0$,

Where $\partial V =$ change of velocity, $\partial S =$ length of flow in the direction S.

On the other hand the flow in which the velocity at any given time changes with respect to space is called non-uniform flow. Thus, mathematically,

$$\left(\frac{\partial V}{\partial S}\right)_{t=constant} \neq 0 .$$

Rotational and Irrotational flows

Rotational flows is defined as the flow in which the fluid particles while flowing along stream lines, also rotate about their own axis and if the fluid particles while flowing along stream lines, do not rotate about their own axis then that type of flow is called irrotational flow.

One, two and three- dimensional flows

A flow field is best characterized by the velocity distribution and thus a flow is said to be one, two or three-dimensional if the flow velocity varies in one, two or three primary dimensions respectively. A typical fluid flow involves a three-dimensional geometry and the velocity may vary in all three dimensions, rendering the flow is three-dimensional [$V(x, y, z)$ in rectangular or $V(r, \theta, z)$ in cylindrical coordinates]. However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored with negligible error. In such cases, the flow can be modeled conveniently as being one or two-dimensional, which is easier to analyze.

Drag and Lift

When a fluid moves over a solid body, it exerts pressure forces normal to the surface and shear forces parallel to the surface along the outer surface of the body. We are usually interested in the resultant of the pressure and shear forces acting on the body rather than the details of the distribution of these forces along the entire surface of the body. The component of the resultant pressure and shear forces that acts in the flow direction is called the drag force (or just drag) and the component that acts normal to the flow direction is called the lift force (or just lift)

Since the measurement of drag and lift depend on the transition in boundary layer, separation of the boundary layer and so on, it is a very difficult task to measure them. We therefore, employ experimental data and define them as follows:

$$Drag = \frac{1}{2} C_D A \rho U^2 ,$$

$$\text{and } Lift = \frac{1}{2} C_L A \rho U^2 ,$$

Where C_D is the coefficient of drag, C_L is the coefficient of lift, A is the projected area in the direction of flow and U is the free-flow velocity.

1.2 Development of Boundary Layer Theory:

The concept of boundary layer theory of fluid flows for large Reynolds number or small viscosity was propounded by **L. Prandtl (1904)**. The theory unified the two divergent branches of fluid dynamics, namely inviscide hydrodynamics and hydraulics and gave quite agreeable results for drag on the solid body around which the fluid moves as compared to the results obtained by the Stokes method of neglecting the non-linear inertia terms in Navier-Stokes equation. He established through theoretical considerations and several simple experiments that the flow about a solid body can be divided into two regions: a very thin layer in the neighbourhood of the body called the boundary layer, where friction i.e. the viscous force plays an essential part and the remaining region outside the boundary layer, where friction may be neglected and the flow there may be regarded as inviscide and irrotational. Thus the tangential (shearing) stress and the condition of no slip at solid walls which distinguishes a real fluid from a perfect fluid are to be taken into consideration only in the boundary layer.

1.2. I Velocity Boundary Layer:

To understand the concept of velocity boundary layer, one has to consider flow over the flat plate of fig. 1.2. (i). when fluid particles make contact with the surface, they assume zero velocity. These particles then act to retard the motion of particles in the adjoining fluid layer, which act to retard the motion of particles in the next layer and so on until at a distance $y = \delta$ from the surface, the effect becomes negligible. This retardation of fluid motion is associated with shear stresses τ acting in planes that are parallel to the fluid velocity (fig. 1.2.(i)). With increasing distance from the surface, the x velocity component of the fluid u, must then increase until it approaches the free stream value u_∞.

The quantity δ is termed as the velocity boundary layer thickness and it is typically defined as the value of y for which $u = 0.99u_\infty$. The boundary layer velocity profile refers to the manner in which u varies with y through the boundary layer. Accordingly, the fluid flow is characterized by two distinct regions, a thin fluid layer (the boundary layer) in which velocity gradients and shear stresses are large and a region outside the boundary layer in which velocity gradients and shear stresses are negligible. With increasing distance from the leading edge, the effects of viscosity penetrate further into the free stream and the boundary layer grows.

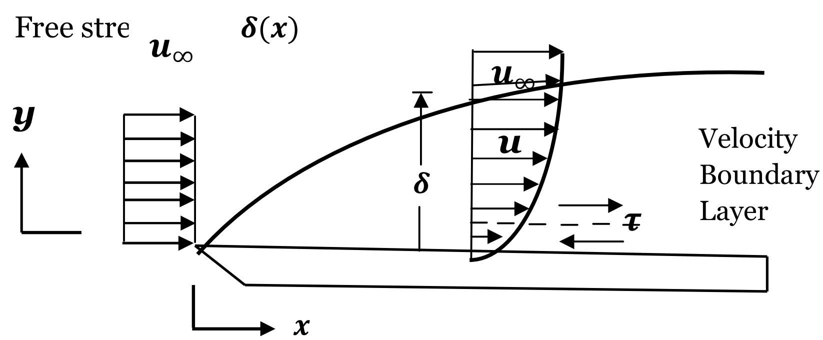

Figure 1.2 (i): Velocity boundary layer developments on a flat plate.

Because it pertains to the fluid velocity, the foregoing boundary layer may be referred to more specifically as the velocity boundary layer. It develops whenever there is fluid flow over a surface and it is of fundamental importance to problems involving convection transport.

1.2. II Thermal Boundary Layer:

Just as the velocity boundary develops when there is fluid flow over a surface, a thermal boundary layer must develop if the fluid free stream and surface temperatures differ. To explain this concept, we consider flow over an isothermal flat plate as shown in Fig. 1.2 (ii). At the leading edge the temperature profile is uniform, with $T(y) = T_\infty$. However fluid particles that come into contact with the plate achieve thermal equilibrium at the plate's surface temperature. In turn these particles exchange energy with those in the adjoining fluid layer and temperature gradients develop in the fluid. The region of the fluid in which these temperature gradients exist is the thermal boundary layer and its thickness δ_t is typically defined as the value of y for which the ratio $(T_s - T)/(T_s - T_\infty) = 0.99$. With increasing distance from the leading edge, the effects of heat transfer penetrate further into the free stream and the thermal boundary layer grows.

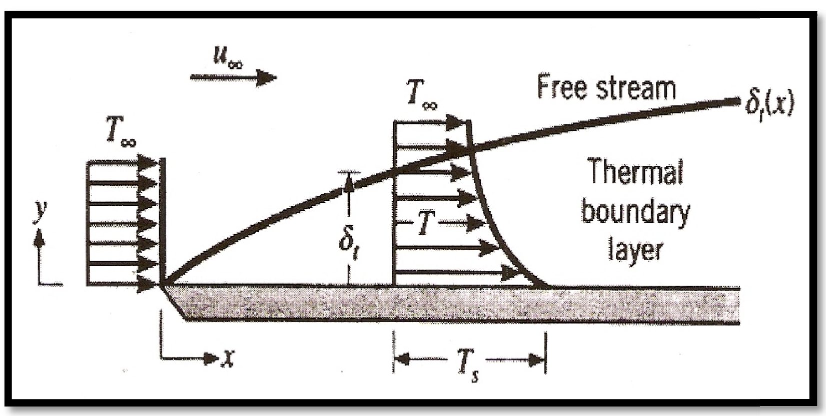

Figure: 1.2 (ii) Thermal boundary layer development on an isothermal flat plate.

1.2. III The Concentration Boundary Layer:

Just as the velocity and thermal boundary layers determine wall friction and convection heat transfer, the concentration boundary layer determines convection mass transfer. If a binary mixture of chemical species A and B flows over a surface and the concentration of species A at the surface $C_{A,S}$, differs from that in the free stream $C_{A,\infty}$, Fig. 1.2.(iii) a concentration boundary layer develops. It is the region of the fluid in which concentration gradient exist and its thickness δ_c is typically defined as the value of y for which $[(C_{A,S} - C_A) / (C_{A,S} - C_{A,\infty})] = 0.99$. Species transfer by convection between the surface and the free stream fluid is determined by conditions in this boundary layer.

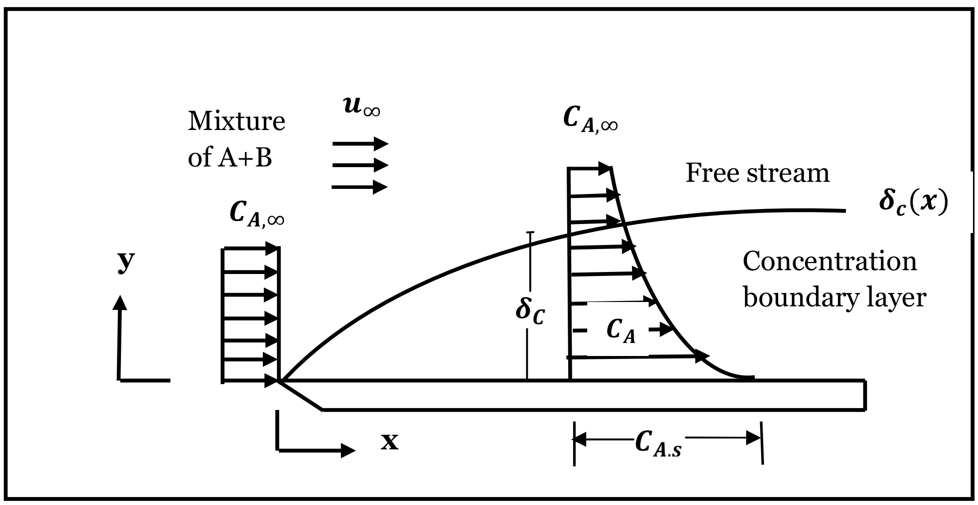

Figure: 1.2 (iii): Species concentration boundary layer development on a flat plate

1.2. IV The No-Slip Condition:

Fluid flow is often confined by solid surfaces and it is important to understand how the presence of solid surfaces affects fluid flow. Here we have to consider the flow of a fluid in a stationary pipe or over a solid surface that is non-porous (impermeable to the fluid). All experimental observations show that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid “sticks” to the surface due to viscous effects and there is no slip. This is known as the no-slip condition. This no-slip condition is responsible for the development of the velocity profile. The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the boundary layer. The fluid property responsible for the no-slip condition and the development of the boundary layer is viscosity.

A fluid layer adjacent to a moving surface has the same velocity as the surface. A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface. Another consequence of the no-slip condition is the surface drag, which is the force a fluid exerts on a surface in the flow direction.

1.2. V Skin-friction Coefficient:

In fluid mechanics, the significance of velocity boundary layer to the engineer, stems from its relation to the surface shear stress and hence to surface frictional effects. For external flows it provides the basis for determining the local frictional coefficient

$$C_f = \frac{\tau_s}{\rho\, u_\infty^2/2} \,, \tag{1.2.1}$$

Where ρ being the fluid density.

A key dimensionless parameter from which the surface frictional drag or skin-friction may be determined.

On the other hand, the shear stress within the boundary layer is of appreciable amount even for fluids with small viscosity owing to large velocity gradient across the flow. The velocity gradient $\left({\partial u}/{\partial y}\right)$ gradually diminishes from its maximum value at the wall to a negligible value at the edge of the boundary layer. The shear stress at the surface is given by

$$\tau = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} , \tag{1.2.2}$$

Where μ is a fluid property known as coefficient of viscosity.

1.2. VI Heat Transfer:

Heat transfer is a spontaneous irreversible process that takes place between material bodies as a result of temperature difference. The transfer of heat between a solid body and a liquid or gaseous fluid is a problem which involves fluid motion. This transport can take place in three different modes: Conduction, Convection and radiation.

Conduction

Heat conduction is identified as process of molecular transport of heat in bodies (or between them), due to temperature variation in the medium concerned. Conduction, in general is the sole agent which transports energy within a solid material. It is the process in which heat is transferred from regions of higher temperature within a system or between two systems which are in contact physically without any relative motion of the different parts of the system or systems. In fact energy is conducted through a material in which a temperature gradient exists by the thermal motion of various microscopic particles of which the material is composed.

Convection

Heat transfer by convection is due to fluid motion. A heat transfer occurring in fluid motion, in which the diffusion of thermal energy is affected by relative motion within the fluid, is called convection. Convection is possible only in the fluid medium. Heat transfer by convection is always accompanied by conduction. The combined process of heat transfer by convection and conduction is referred to as convective heat transfer. It is the process of heat transfer whose rate is directly influenced by the fluid motion. Thus the heat may be finally transferred through the flowing material by conduction, but the conduction process is basically altered by relative motion of the microscopic particles in the fluid. Thermal energy and mass are convected about the flow region by the motion of the fluid.

Moreover, heat is transferred by conduction and convection in a system of fluid motion. It is quite evident from the method of heat transfer from a surface to the surrounding fluid. At first heat is transferred from the surface by conduction to the adjacent fluid elements which in turn move to regions of lower temperature and impart heat to the neighbouring fluid particles by conduction as well. Thus convective process dominates a heat transfer phenomenon in fluid mechanics. As the convective heat transfer process and the motion of the fluid are inseparable, a study of hydrodynamic behaviour of the fluid is necessary in order to understand heat transfer taking place within a moving fluid.

Free and Forced Convection

The problem of thermal convection may be subdivided into two groups namely forced convection and free or natural convection.

Forced Convection

A convective process which takes place due to velocities created by an external agency such as forcing a fluid past some solid object is termed as forced convection i.e. the convection transfer in fluid flows that originate from an external forcing condition. This happens at large velocities (at large Reynolds number) and small temperature differences. For example, fluid motion may be induced by a fan or a pump or it may result from propulsion of a solid through the fluid.

Free Convection

A convective process caused by the action of body forces such as gravitation on the fluid, which arises as a result of density gradients due to changes in temperature, is termed as free convection (Natural convection). These density gradients give rise to distributed buoyancy force, which causes relative motion. Hence in free convection flow, the velocity and the temperature fields are coupled.

In general the free convection flow velocities are much smaller than those associated with forced convection, the corresponding convection transfer rates are also smaller. In many systems involving multimode heat transfer effects, free convection provides the largest resistance to heat transfer and therefore plays an important role in the design or performance of the system. Moreover, when it is desirable to minimize heat transfer rates or to minimize operating cost, free convection is often preferred to forced convection.

There has recently been a considerable interest in the effect of body forces on forced convection phenomena. In certain engineering problems, however they cannot be left out of consideration. It is important to realize that the heat transfer in mixed convection can be significantly different from that both in pure natural convection and in pure forced convection.

The study of forced and free convection flow and heat transfer for electrically conducting fluids past a semi-infinite porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as geophysics, astrophysics, boundary layer control in the field of aerodynamics. Engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems etc. Because of the above practical importance of such problems many researchers have been working in this field and some of these are mentioned here. Soundalgekar (1973) obtained approximate solutions for the two-dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate, the difference between the temperature of the plate and the free stream is moderately large causing the free convection currents. The natural convection heat transfer from an isothermal vertical wavy surface was first studied by Yao (1983) and using an extended Prantdl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surface. Moulic and Yao (1989) also investigated mixed convection heat transfer along a vertical wavy surface. Alam *et al*. (1997) have also studied the problem of free convection from a wavy vertical surface in the presence of a transverse magnetic field. Combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees (1999). Sakiadis (1961) was the first author to analyze the boundary layer flow on a continuous surface. Gorla *et al.* (1988, 1987) solved the non similar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature. Cheng and Minkowycz (1977) also studied free convection from a vertical flat plate with applications to heat transfer from a dick. The unsteady free convection flow past an infinite porous plate and semi-infinite plate were studied by Nanda and Sharma (1962). In their first paper they assumed the suction velocity at the plate varying in time as $t^{-\frac{1}{2}}$, where as in the second paper the plate temperature was assumed to oscillate in time about a constant nonzero mean. Free convective flow past a vertical plate has been studied extensively by Ostrach (1953) and many others. The free convective heat transfer on vertical semi-infinite plate was investigated by Berezovsky *et al*. (1977). Martynenko *et al.* (1984) investigated the laminar free convection from a vertical plate. Muthucumaraswamy and Meenakshisundaram (2006) investigated theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature and mass diffusion. Shrama *et al.* (2010) investigated the effect of temperature dependent electrical conductivity on steady natural convection flow of a viscous incompressible low Prandtl ($Pr << 1$) electrically conducting fluid along an isothermal vertical non-conducting plate in the presence of transverse magnetic field and exponentially decaying heat generation. Cheng and Hsu (1988) studied the steady forced convection problem in packed sphere beds. Makinde *et al.* (2010) studied the unsteady flow and heat transfer of a dusty fluid between two parallel plates with variable viscosity and electric conductivity.

Radiation

The mode of heat transfer that takes place in the form of electromagnetic waves is called radiation. It depends only on the temperature and on the optical properties of an emitter, with its internal energy being converted into radiation energy. The process of conversion of the internal energy of a substance into radiation energy is referred to as radiation heat transfer. Radiant emission is also due to thermal motion of microscopic particles, but the energy is transmitted electro-magnetically. The laws of radiation are as follows

Stefan-Boltzmann's Law

It states that the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the surface of the body i.e.

$$E \propto T^{*4} \quad \text{or} \quad E = \sigma^* T^{*4},$$

Where σ^* is called the Stefan's constant. Its value is

$$5.67 \times 10^{-5} \quad ergs\, s^{-1}\, cm^{-2} K^4 \quad in\ C.G.S.\ system \quad \text{or}$$

$$5.67 \times 10^{-8} JS^{-1} m^{-2} K^4 \quad or \quad 5.67 \times 10^{-8} Wm^{-2} K^{-4} \quad \text{in S.I.}$$

Kirchhoff's Law of Radiation

Kirchhoff's law states that the ratio of the emissive power and the absorptive power for radiation of a particular wavelength and at a particular temperature is constant for all bodies. This ratio is also equal to the emissive power of a perfectly black body at the temperature i.e.

$$\frac{E_\ell}{1} = \frac{Q}{d_\ell} \quad and \quad \frac{e_\ell}{a_\ell} = E_\ell = \text{constant},$$

Where e_ℓ, a_ℓ and Q are emissive power, absorptive power and quantity of heat radiation incident on the surface respectively.

Planck's Law of Radiation

Planck introduced the quantum concept in 1900 and with it the idea that radiation is emitted not in a continuous energy state but in discrete amounts or quanta. The intensity of radiation emitted by a black body, derived by Planck is

$$I_{b,\ell} = \frac{2c^2 h' \ell^{-5}}{exp\left(\frac{ch'}{\kappa \ell T^*}\right) - 1},$$

Where $I_{b,\ell}$ is the intensity of radiation from a black body between wavelengths ℓ and $\ell + d\ell$, c is the speed of light, h' is Planck's constant, κ is the Boltzmann constant and T^* is the temperature. The total emissive power between wavelengths ℓ and $\ell + d\ell$ is then

$$E_{b,\ell} = \frac{2\pi c^2 h' \ell^{-5}}{exp\left(\frac{ch'}{\kappa \ell T^*}\right) - 1}.$$

The radiative flows have lots of significant applications in industrial and environmental processes e.g. heating and cooling chambers, fossil fuel combustion and energy processes evaporation from large open water reservoirs and solar power technology. In view of these uses many researchers investigated it and some of these literatures have been enlisted here. Vasu *et al.* (2011) studied radiation and mass transfer effects on transient free convection flow of dissipative fluid past a semi-infinite vertical plate with uniform heat and mass flux. Again radiation and mass transfer effects on free convection flow through porous medium bounded by a vertical surface were examined by Raju *et al.* (2011). The radiation effects on boundary layer flow with and without applying a magnetic field under different situations has been studied by many investigators, for examples: Israel-cookey *et al*. (2003), Mahmoud (2007), Hayat *et al.* (2007). England and Emery (1969) studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Prasad *et al.* (2010) studied the radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in porous medium. Ibrahim *et al*. (2012) proposed the radiation and chemical reaction effects on MHD free convection flow past a moving vertical plate. Abd El-Naby *et al*. (2003) numerically studied magnetohydrodynamic (MHD) transient natural convection-radiation boundary layer flow with variable surface temperature, showing that velocity, temperature and skin friction are enhanced with a rise in radiation parameter increases, whereas Nusselt number is reduced. Ogulu and Prakash (2006) obtained analytical solutions for variable suction and radiation effects on dissipative-free convective, optically-thin, magnetohydrodynamic flow using a differential approximation to describe the radiative flux. Moreover, studies involving thermal radiation and transient hydromagnetic convection include the analyses by Prasad *et al*. (2006) which included species transfer and Zueco (2007) who also considered viscous heating. Mebine (2011) studied the effects of thermal radiation on transient MHD free convection flow over a vertical surface embedded in a porous medium with periodic temperature and obtained analytical solutions for the governing coupled dimensionless partial differential equations of velocity and temperature. Miraj *et al*. (2011) studied conjugate effects of radiation and joule heating on magnetohydrodynamic free convection flow along a sphere with heat generation. Ghaly (2002) considered the thermal radiation effect on a steady flow, whereas Rapits and Massalas (1998). Ferdows *et al.* (2004) analyzed free convection flow with variable suction in presence of thermal radiation. Ibrahim *et al*. (2008) studied the effects of chemical reaction and radiation absorption on transient hydromagnetic natural convection flow with wall transpiration and heat source. Analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generation/ absorbing fluid on a continuously vertical permeable surface in the presence of a radiation, a first – order homogeneous chemical reaction and the mass flux are reported by Kesavaiah *et al.* (2011). Korycki (2006) described radiative heat transfer as an important fundamental phenomena existing in practical engineering such as those found in solar radiation in buildings, foundry engineering and solidification processes, chemical engineering, composite structures applied in industry. Rashidi *et al*. (2014) examined free convective heat and mass transfer in a steady two-dimensional magnetohydrodynamic fluid flow over a stretching vertical surface in porous medium and in this study thermal radiation and non-uniform magnetic field were taken into consideration.

1.3 Magnetohydrodynamics (MHD):

Magnetohydrodynamics (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic field. Many natural phenomena and engineering problems are worth being subjected to an MHD analysis. MHD equations are ordinary electromagnetic and hydrodynamic equations modified to take into account the interaction between the motion of the fluid and the electromagnetic field. The formulation of the electromagnetic theory in mathematical form is known as Maxwell’s equation. The effect of the gravity field is always present in forced flow heat transfer as a result of the buoyancy forces connected with the temperature differences. Usually they are of a small order of magnitude so that the external forces may be neglected.

In the last few decades, much work has been done on the generalization of viscous flow and heat transfer solutions to take account of the additional effects of a magnetic field when the fluid involved is electrically conducting. It was known from Faraday's (1832) time that a solid body on a fluid material moving in a magnetic field experiences on electromotive force (e. m. f.). If the material is electrically conducting and a current path is available, electric current ensure. Also currents may be induced by change of the magnetic field with time. There are two basic consequences:

I. An induced magnetic field associated with these currents appears, perturbing the original magnetic field.

II. An electromotive force due to the interaction of currents and field appears, perturbing the original motion.

The motion of fluid velocity affects the magnetic field by carrying the magnetic fields lines partially (depending upon the electrical conductivity of the fluid) along with it and the magnetic field affects the motion by producing a mechanical force namely, the Lorentz force $\vec{J} \times \vec{B}$, where $\vec{J}$ is electric current density and $\vec{B}$ the magnetic induction vector in the fluid region.

The study of MHD is quite important in the field of Aeronautics, especially Missile Aerodynamics, since the temperature that occur in such fluid speeds are sufficient to dissociate or even ionize the air appreciably. For example, when a high speed missile re-enters the earth's atmosphere, a very large amount of heat is generated due to the friction of air molecules as this viscous heating may sometimes be so considerable as to ionize the air near the forward stagnation point. Again, for most of the liquids and gasses are poor conductors of electricity. As a consequence their motion can normally be treated by the principles of fluid dynamics. However, it is possible to make some gasses very highly conducting by ionizing them. For ionization to take effect, the gas must be very hot at temperatures upwards of 5000^0K or so. Such ionized gasses are called PLASMA. The material within a star is plasma of very high conductivity will exists within a strong magnetic field. Astro-physicians come into realize that the whole universe are conducting, ionized gasses (plasmas) and significantly strong magnetic fields. In the interwar period the astrophysicists, notably Cowling (1934) and Ferraro (1957), began to explore the formal theory of MHD and its applications, while other scientist and engineers such as William (1930) and Hartmann (1937) performed simple experiments on the flow of conducting liquids in the laboratory.

In MHD heat transfer problems, the additional body force term viz. the ***Lorentz force*** comes into play in the momentum equation and the term corresponding to Joule heating appears in the energy equation. In a forced convection system, the energy equation remains uncoupled from Maxwell's equations and Navier-Stokes equations. Thus the electromagnetic and velocity fields can be determined independently of the temperature field. However, when natural convection forces are present, the Navier-Stokes equation becomes coupled with the energy equation and simultaneous solution is required. In view of natural convection problems, the velocity being zero in the free stream; the induced magnetic field does not exist there. Thus the influence of the magnetic field on the boundary layer is extended through the *Lorentz force* confined to the boundary layer only, with no additional effects arising out of the free stream pressure gradient. Thus the free convection MHD problems can be formulated in a much simpler way than the corresponding forced convection problems.

Magnetohydrodynamic flows in porous media have stimulated considerable attention owing to the importance of such flows in magnetic materials processing (1977), chemical engineering (1989) and geophysical energy systems (1994). Considering these applications of the flow through porous medium, a series of investigation has been performed by different researchers. For instance Chaudhary and Jain (2008) studied the influence of oscillating temperature on magnetohydrodynamic convection heat transfer past a vertical plane in a Darcian porous medium. Rossow (1958), Greenspan and Carrier (1959) studied extensively the hydromagnetic effects on the flow past a plate with or without injection/suction. The hydromagnetic channel flow and temperature field was investigated by Attia and Kotab (1996). Steven *et al*. (2012) studied the magnetohydrodynamic free convective flow past an infinite vertical porous plate with the effect of viscous dissipation subject to a constant suction velocity. Singer (1965) further assessed the unsteady free convection heat transfer with magnetohydrodynamic effects in a channel regime. Rao (1971) analyzed the unsteady magnetohydrodynamic convection heat transfer past an infinite plane. Soundalgekar *et al.* (1973) studied on fully-developed MHD free convective flow between two vertical, electrically conducting plates and observed significant result on Hartmann number, thermal conductance ratio and line heat source. Ram (1991) investigated the steady magnetohydrodynamic convective flow of a partially ionized gas past an infinite vertical porous plate in a rotating frame of reference taking Hall and Ion-slip currents into account and discussed the effect of Hall and ion-slip currents as well as the other parameter entering into the problem. Chaudhary and Jain (2008) presented an analytical study of magnetohydrodynamic transient convection flow past a vertical surface embedded in a porous medium with an oscillating temperature. Alam *et al*. (2006) studied Dufour and Soret effect with variable suction on unsteady MHD free convection flow along a porous plate. Mishra *et al*. (2013) investigated free convective fluctuating MHD flow through porous media past a vertical porous plate with variable temperature. Sarada and Shanker (2013) studied the effect of chemical reaction on an unsteady magneto hydrodynamic flow past an infinite vertical porous plate with variable suction and heat convective mass transfer, where the plate temperature oscillates with the same frequency as that of variable suction velocity. The non–linear partial differential equations governing the flow have been solved numerically using finite difference method. Ahmed and Batin (2014) investigated the flow model of steady free convective MHD flow of an incompressible viscous electrically-conducting fluid over an infinite vertical isothermal porous plate with mass convection. Chen (2004) employed a numerical method to study the heat and mass transfer in MHD free convective flow with Ohmic heating and viscous dissipation. Turkyilmazoglu (2011) presented multiple solutions in visco-elastic MHD fluid flow and heat and mass transfer over stretching and shrinking surfaces.

1.4 Porous Medium:

In 1962, Leonhard Euler's description of a porous body was worth mention. His remarks on porous bodies are: "All bodies the World are composed of rough and suitable matter; the first one is called the characteristics matter whereas the other due to its real infinity small density contributes nothing to the increase of their mass. Since the mixture of both matters extends to the smallest part, those parts of the space, in which no rough matter contained, are called the pores of the bodies, and there are different kinds concerning the size, because also the smallest parts are still filled up with pores. The most distinct difference however, which must be considered for the pores of any body, is that some form of an open part with the others, whereas other one are surrounded by the rough matter in such a way that the sub tile matter there in contained cannot escape. In order to denote these differences we call the first open pores and the last closed pores". In 1760 he guessed an example of this definition, the water saturated porous, solid which is of immediate relevance to the topic. Reinhard Woltman (1794) was a harbor construction director (1757-1837) from Hamburg, he expanded his idea on soil machines and porous bodies and introduced the volume fraction concept, an essential part of the theory of porous medium. Around the mid 19^{th} century, fundamental effects concerning porous medium were studied and described by Delesse, Fick and Darcy (1996), namely the equality of surface and volume fractions in porous medium with statistically distributed pores, the diffusion phenomenon, in the interaction between the constituents.

Darcy (1856) was the first scientist to study the interaction between two constituents in between the skeleton (Porous soil body) and water. He observed, in tests with natural sand, the proportionality of the total volume of water running through the sand and the loss of pressure. Although these investigations were of a purely experimental nature, his results are essential for a continuous mechanical treatment of the motion of a liquid in a porous solid. The porous medium is in fact a non-homogeneous medium. For the sake of analysis, it is possible to describe the flow in terms of a homogeneous fluid with averaged dynamic properties having some effects on the locally non-homogeneous continuum. Thus the flow problems of non-homogeneous fluid under the action of the properly averaged external forces can be studied. On the basis of this hypothesis a complicated problem of flow through a porous medium reduces to the flow problem of a homogeneous fluid with some resistance. Today Darcy's law is theoretically well founded by thermodynamics. For a homogeneous medium the Darcy's law is expressed in vector form as

$$\vec{q} = -\frac{\vec{K}}{\mu}.\vec{\nabla}P\ , \tag{1.4.1}$$

Where $\vec{K}$ is in general a second order tensor.

For the case of an isotropic medium the permeability is a scalar and the equation (1.4.1) simplifies to

$$\vec{\nabla}P = -\frac{\mu}{K}\vec{q} \tag{1.4.2}$$

Following Wooding (1957), many early authors on connection in porous medium used an extension of equation (1.4.2) of form

$$\rho\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\vec{\nabla}p - \frac{\mu}{K}\vec{q} \tag{1.4.3}$$

An alternative to Darcy's equation what is commonly known as Brinkman's equation and is

$$\vec{\nabla}P = -\frac{\mu}{K}\vec{q} + \tilde{\mu}\nabla^2\,\vec{q}\ , \tag{1.4.4}$$

with inertial term omitted.

Combining the equations (1.4.3) and (1.4.4) together the Navier-Stokes equation of motion for any incompressible viscous fluid through a porous medium can be written as follows:

$$\rho\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\vec{\nabla})\vec{q}\right] = \rho\vec{F} - \vec{\nabla}P + \mu\nabla^2\vec{q} - \frac{\mu}{K}\vec{q}\ , \tag{1.4.5}$$

Where $\vec{F}$ is the external force acting in the fluid per unit mass.

Fluid flow through a porous media has been studied theoretically and experimentally by numerous authors due to its wide applications in various fields and some of these are listed here. An analytical solution for unsteady free convection in porous media has been studied by Magyari *et al*. (2004). Chamkha *et al*. (2000) studied the effects of Hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. Influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source was investigated by Sharma *et al*. (2011). Mahapatra *et al*. (2010) studied the effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Sattar (1992) studied numerically free convection flow through a porous medium bounded by a semi-infinite vertical porous plate and obtained analytical solution by the perturbation technique adopted by Singh and Dikshit (1998). Sattar *et al.* (2000) studied unsteady free convection flow along a vertical porous plate embedded in a porous medium. Ahmed (2007) looked the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Chaudhary and Jain (2009) discussed the MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium. Radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi – infinite vertical permeable moving plate embedded in a porous medium was studied by Prasad *et al.* (2008). Ramana Reddy *et al*. (2010) investigated the mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption. Patil and Kulkarni (2008) studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Ahmed *et al.* (2014) investigated for the model of unsteady MHD thermal convection flow of a viscous incompressible absorbing-emitting optically thin gray gas along an impulsively-started semi-infinite vertical plate adjacent to the *Darcian* porous regime in presence of a first order chemical reaction and significant thermal radiation effects. The study of the thermal radiation and chemical reaction effects on an unsteady MHD free convective mass transfer flow past an accelerated infinite vertical plate embedded in a porous medium was performed by Sarma *et al.* (2014). The effects of chemical reaction and thermal starification over a vertical stretching surface in a porous medium were considered by Mansour *et al*. (2008).

Acharya *et al*. (2014) studied free convective magnetohydrodynamics (MHD) flow of a viscous incompressible and electrically conducting fluid past a hot vertical porous plate embedded in a porous medium in the presence of heat source and they recorded that the presence of porous media has no significant contribution to the flow characteristics and viscous dissipation compensates for the heating and cooling of the plate due to convective current. Sarada and Shankar (2013) studied the numerical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting fluid on a continuously vertical permeable porous surface in the presence of a heat source, a first order homogeneous chemical reaction and the mass flux. Leong *et al.* (2005) studied on heat transfer of oscillating flow through a channel filled with aluminum foam subjected to a constant wall heat flux. The surface temperature distribution on the wall, velocity of flow through porous channel and pressure drop across the test section were measured. Hassanien *et al.* (1990) investigated about a problem of two-dimensional unsteady flow of a viscous, incompressible, electrically-conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its own plane. Kaviany (1985) used a numerical solution of laminar flow in a porous channel bounded by isothermal parallel plates and his work was based on the Darcy model. Poulikakos and Renken (1987) used a variable porosity model and numerically investigated the effects of flow inertia bounded by parallel plates and also for circular tubes. Barletta *et al.* (2007) studied on fully developed laminar mixed convection flow in a vertical plane parallel channel filled with a porous medium and subject to isoflux-isothermal wall conditions is investigated assuming that (i) the Darcy law and the Boussinesq approximation hold, (ii) the effect of viscous dissipation is significant. Kim *et al.* (1994) investigated a numerical study which is made of heat transfer characteristics from forced pulsating flow in a channel filled with fluid-saturated porous media. The channel walls were assumed to be at uniform temperature. The Brinkman-Forchheimer-extended Darcy

model was employed. The time-dependent, two-dimensional governing equations were solved by using finite-volume techniques. Vafai and Kim (1989) studied porous forced convection between two parallel plates. Hooman *et al.* (2007) investigated numerically the forced convection with viscous dissipation in a parallel plate channel filled by a saturated porous medium.

1.5 Governing Equations:

The basic equations governing the motion of an incompressible, viscous and electrically conducting fluid through a porous medium in the presence of a magnetic field, heat sources and sinks are as follows:

I. Equation of continuity:

$$div\,\vec{q} = 0 \,, \tag{1.5.1}$$

II. Equation of motion (modified Navier-Stokes equation) :

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = \vec{F} - \frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} + \vec{J}\times\vec{B} - \frac{\nu}{K}\vec{q}\,, \tag{1.5.2}$$

III. Equation of energy:

$$\rho C_p\left[\frac{\partial T}{\partial t} + (\bar{q}.\nabla)T\right] = \kappa\nabla^2 T + \mu\Phi - \frac{\bar{J}^2}{\sigma} + Q\,, \tag{1.5.3}$$

IV. Equation of mass transfer (modified):

$$\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = \nabla^2 C\,, \tag{1.5.4}$$

where in rectangular Cartesian coordinates

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{1.5.5}$$

$$\Phi = 2\left[\begin{array}{c}\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right\} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \\ + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 - \frac{2}{3}\left\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right\}^2\end{array}\right] \tag{1.5.6}$$

Maxwell's equations in rationalized MKS system of unit are:

$$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1.5.7}$$

$$\text{curl}\vec{B} = \mu_e \vec{J} \tag{1.5.8}$$

$$\text{Div}\,\vec{B} = 0 \tag{1.5.9}$$

$$\vec{J} = \left[\vec{E} + \vec{q} \times \vec{B}\right] \tag{1.5.10}$$

1. 6 Boundary Conditions:

The boundary conditions of a flow of an incompressible, viscous, electrically conducting fluid through a porous medium in presence of a transverse magnetic field are:

(I) there is no slip of fluid on the boundary ;

(II) $T = 0$ or $\frac{\partial T}{\partial n} = 0$ or $T = T_W$ on the boundary ;

(III) $T \to T_\infty$ at a large distance from the boundary ;

(IV) $C \to C_\infty$ at a large distance from the boundary ;

(V) the normal component of the magnetic induction is continuous across the interface

(VI) If none of the regions (fluid, solid or vacuum) is perfectly conducting, the tangential component of the magnetic field $\vec{H} = \frac{\vec{B}}{\mu_e}$ is continuous across the interface. If, however, at least one of two media in contact is perfectly conducting, then the magnetic field $\vec{H} = \frac{\vec{B}}{\mu_e}$ must satisfy the condition

$$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_S \ ,$$

Where $\vec{n}$ is the unit vector normal to the surface, $\vec{H}_1, \vec{H}_2$ are the values of the magnetic field on two sides of the interfaces and $\vec{J}_S$ is the surface current density.

(VII) The tangential component of the electric field is continuous across the interface.

The equation of continuity, motion and energy can be simplified with the usual boundary layer approximations whenever a problem of boundary layer flow and heat transfer is considered.

1.7 Non-dimensional Quantities:

The non-dimensional quantities are introduced as follows:

Reynolds Number (Re)

The Reynolds number is defined as

$$Re \equiv \frac{Inertia\ force}{Viscous\ force} = \frac{U_\infty L}{\nu}$$

It is the most important dimensionless number in fluid dynamics providing a criterion for dynamic similarity. The Reynolds number is used for determining whether a flow is laminar or turbulent.

Prandtl Number (Pr)

It is a measure of the relative importance of heat conduction and viscosity of the fluid. The Prandtl number, like the viscosity and thermal conductivity, is a material property and it thus varies from fluid to fluid. Usually Prandtl number is large when thermal conductivity is small and viscosity is large and small when viscosity is small and thermal conductivity is large. It is defined as

1.7 Non-dimensional Quantities:

The non-dimensional quantities are introduced as follows:

Reynolds Number (Re)

The Reynolds number is defined as

$$Re \equiv \frac{Inertia\ force}{Viscous\ force} = \frac{U_\infty L}{\nu}$$

It is the most important dimensionless number in fluid dynamics providing a criterion for dynamic similarity. The Reynolds number is used for determining whether a flow is laminar or turbulent.

Prandtl Number (Pr)

It is a measure of the relative importance of heat conduction and viscosity of the fluid. The Prandtl number, like the viscosity and thermal conductivity, is a material property and it thus varies from fluid to fluid. Usually Prandtl number is large when thermal conductivity is small and viscosity is large and small when viscosity is small and thermal conductivity is large. It is defined as

$$\Pr \equiv \frac{Kinematic\ viscosity}{Thermal\ diffusivity} = \frac{\nu}{a} = \frac{\mu C_p}{\kappa}.$$

Grashoff Number (Gr)

The Grashoff number usually occurs in free convection problems. This gives the relative importance of buoyancy force to the viscous forces. This number is defined as

$$Gr = \frac{Buoyancy\ force}{Viscous\ force} \equiv \frac{g\ \beta\ (\Delta T)_0\ L^3}{\nu^2}$$

Modified Grashoff Number (Gm)

The modified Grashoff number usually occurs in free convection problems, when the effect of mass transfer is also considered. This number is defined as

$$Gm \equiv \frac{g\,\bar{\beta}\,(\Delta C)_0\,L^3}{\nu^2}$$

Eckert Number (Ec)

The Eckert number is defined as

$$Ec = \frac{Kinetic\ energy}{Enthalpy} \equiv \frac{U_\infty^2}{C_p(\Delta T)_0}$$

In compressible fluids it determines the relative rise in temperature of the fluid due to adiabatic compression.

Hartmann Number (M)

The Hartmann number is defined as

$$M = \frac{Magnetic\ body\ force}{Viscous\ force} \equiv \frac{\sigma\,B_0^2\nu}{\rho\,U_\infty^2}$$

Nusselt Number (Nu)

The dimensionless coefficient of rate of heat transfer which is generally known as the Nusselt number, is defined as

$$Nu = \frac{Convection\ heat\ transfer}{Conduction\ heat\ transfer} \equiv \frac{Lh}{\kappa} \equiv -\frac{L}{(\Delta T)_0}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Sherwood Number (Sh)

The dimensionless coefficient of rate of mass transfer which is generally known as the Sherwood number, is defined as

$$.Sh = \frac{Overall\ mass\ diffusion}{Species\ diffusion} \equiv \frac{VL}{D_{AB}} \equiv -\frac{L}{(\Delta C)_0}\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Schmidt Number (Sc)

This number is the ratio of momentum diffusivity to molecular diffusivity. It is defined as

$$Sc \equiv \frac{Momentum\ diffusivity}{Molecular\ diffusivity} = \frac{\nu}{D}.$$

The Schmidt number plays a role in convective mass transfer analogous to that of Prandtl number in convective heat transfer.

Magnetic Reynolds number (Rm)

The magnetic Reynolds number is defined as

$$Rm = \frac{Magnetic\ convection}{Magnetic\ diffusion} \equiv UL\sigma\mu_e = \frac{UL}{\eta}$$

If $Rm << 1$, it can be shown that the induced magnetic field is small compared to the applied magnetic field.

Magnetic Prandtl Number (Pm)

The magnetic Prandtl number is defined as

$$Pm = \frac{Vorticity\ diffusion}{Magnetic\ diffusion} \equiv \frac{\nu}{\eta},$$

Drag coefficient (C_D)

The drag coefficient is defined as

$$C_D = \frac{Drag\ force}{Dynamical\ force} \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

Lift coefficient (C_L)

The lift coefficient is defined as

$$C_L = \frac{Lift\ force}{Dynamical\ force} \equiv \frac{F_L}{\frac{1}{2}\rho V^2 A}$$

Non-dimensional Permeability parameter $S \equiv \frac{K}{L^2}$

Non-dimensional heat source / sink parameter $\bar{\alpha} \equiv \frac{Q\ L^2}{\kappa\ (T_W - T_\infty)}$

1.8 Outline of the Thesis:

The present thesis entitled, "*Some flow problems on magnetohydrodynamic free convective heat transfer flow*" deals with the mathematical investigations of various flow problems pertaining to steady/unsteady, free convective, magnetohydrodynamic flows through porous medium in non-rotating as well as rotating systems.

The thesis consists of eight chapters. The preliminary chapter-I deals with the introduction of fluid mechanics and other relevant areas.

The chapter-II is devoted to investigate the combined effects of injection/suction and magnetic field on the Oscillatory MHD flow through porous medium bounded by the horizontal parallel porous plates. Both the stationary plates are subjected to same constant injection / suction velocities. A uniform magnetic field is applied normal to the planes of the plates. A closed form analytical solution is obtained and the effects of different flow parameters on velocity field and skin-friction are discussed with the help of graphs in detail. It is found that, when the

Darcy number (Da) or suction/injection parameter (λ) is increased, the fluid velocity profiles also increased. An increase in Da or λ is found to escalate the shear stress (τ). Possible applications of the present study include laminar aerodynamics, materials processing and thermo-fluid dynamics.

In chapter III, an Analytical solutions for the steady magnetohydrodynamic laminar mixed convection heat and mass transfer flow of viscous electrically conducting fluid past a vertical permeable surface embedded in a *Darcian* porous medium with thermal radiation and chemical reaction effects has been presented. The heat equation includes the terms involving the radiative heat flux, Ohmic dissipation, viscous dissipation and the internal absorption whereas the mass transfer equation includes the effects of chemically reactive species of first-order. The non-linear coupled differential equations are solved analytically by perturbation technique. Validity of the analysis has been performed by comparing the present results with those available in the open literature and a very good agreement has been established. It is observed that the effect of heat absorption is to decrease the velocity and temperature profiles in the boundary layer.

In chapter-IV, a rotating model is developed for a two-dimensional, unsteady, incompressible electrically conducting, laminar free convection boundary layer flow of heat and mass transport in a saturated porous medium, bounded by an infinite vertical porous surface in presence of an applied transverse magnetic field. The porous plane surface and the porous medium are assumed to rotate in a solid body rotation. The vertical surface is subjected to uniform constant suction perpendicular to it and the temperature at this surface fluctuates in time about a non-zero constant mean. The basic equations governing the flow are in the form of partial differential equations and have been reduced to a set of ordinary differential equations. The problem is tackled analytically using classical perturbation technique. Pertinent results with respect to embedded parameters are displayed graphically and tables for the velocity, concentration and skin friction profiles were discussed quantitatively. Applications to the flows of fluids through porous medium bounded by rotating porous systems find many industrial applications particularly in the fields of centrifugation, filtration and purification processes.

The chapter-V is to investigate the effect of magnetic field and radiation on unsteady Magnetohydrodynamic boundary layer flow and heat transfer through a Darcian porous medium bounded by a uniformly moving semi-infinite isothermal vertical plate in the presence of thermal radiation. The flow model is considered as a viscous, incompressible, electrically-conducting Newtonian fluid which is an optically thin gray gas. Suitable transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into ordinary differential equations. Analytical solutions of these equations are obtained by Laplace transform. The effects of Hartmann number (M), porosity parameter (K), thermal radiation parameter (Ra), and Prandtl number (Pr) on flow velocity, fluid temperature, velocity and temperature gradients at the surface are studied graphically. Velocity is reduced with Hartmann number but enhanced with thermal radiation and porosity parameter. Increasing radiation parameter Ra tends to boost the heat transfer rate at the wall. Applications of the study arise in engineering and geophysical sciences like magnetohydrodynamic transport phenomena and magnetic field control of materials processing, solar energy collector systems.

An analysis of periodic heat and mass transport of unsteady hydromagnetic flow past a parabolic started motion of the infinite vertical plate immersed in Darcian porous regime in presence of a first order chemical reaction has been presented in Chapter VI. Here the plate temperature as well as concentration level near the plate are increased linearly with time. The boundary layer conservation equations have been solved by Laplace transforms technique. They satisfy all imposed initial and boundary conditions and reduce to some well-known solutions for Newtonian fluids. The effects of different physical parameters namely Magnetic field parameter, porosity parameter, Prandtl number, Grashoff number, Schmidt number and chemical reaction on the flow velocity, fluid temperature, concentration have been studied graphically. It has been observed that both the velocity and concentration are decreased with increasing values of chemical reaction parameter. But the opposite behaviour has been found for the flow velocity when the values of free convection as well as porosity parameter are increased. Application of magnetic fields to medical science is growing rapidly, with the development of novel magnetic pumps, hydromagnetic separation devices with chemical engineering and geophysical energy systems.

A theoretical model is developed for unsteady MHD laminar viscous thermal convection flow of an optically-thick gray gas flowing over a semi-infinite vertical moving porous plate embedded in a uniform porous medium including the Soret effects and heat generating/absorbing in Chapter VII. The Rosseland diffusion flux approximation is employed to simulate radiative heat transfer contribution. The plate moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. The dimensionless governing equations have been solved analytically by using perturbation technique. The effects of Soret number (S0), heat generation (Q), Rosseland radiation-conduction parameter (R) and magnetic body force (M) on dimensionless velocity (u), temperature (θ), concentration profiles (∅), coefficient of skin-friction (τ), Nusselt number (Nu) and Sherwood number (Sh) are studied graphically. It is found that with the increasing value of Soret number or porosity, the flow velocity profiles tends to accelerated, while heat generation of the fluid decelerated the flow velocity. These results may useful in natural sciences, engineering sciences and in industry.

Finally the chapter VIII deals with the problem of mixed convection flow of an electrically conducting fluid along a vertical plate embedded in a porous medium in the presence of a uniform normal magnetic field, first order chemical reaction and subjected to a periodic suction velocity. The basic equations comprising the balance laws of mass, linear momentum, and energy have been solved analytically using perturbation technique. Graphical results for the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number profiles are illustrated and discussed for various physical parametric values. The results of our study agree well with the previous solutions obtained without mass transfer and chemical reaction. The present study has great significance in different field of science and engineering.

For the present study several books, journals, articles of different researchers, scientists, authors etc. are used for the reference and these are included in the bibliography.