ABSTRACT

In this thesis we have examined Solitons solutions of certain nonlinear differential equations (Nonlinear Schrödinger equation (NLS), Toda Lattice,Klein-Gordon equation, KdV, Sine Gordon equation ,Perturbed Sine-Gordon equation andNavier Stokes equation) and the role they play in various physical systems (such as nonlinear optical lattices, Polyacetylene and Josephson Junctions).

Solitons are large amplitude solitary waves which are known to propagate for very large distances in nonlinear media. Nonlinear equations govern many aspects of our lives. For example the large amplitude waves which we see at every sea shore are Solitons. To understand the properties, behavior of the nonlinear equations they have to be solved exactly. This has been done in our first paper using Z transform. AKNS and ZS have shown that the temporal and spatial evolution of nonlinear differential equations can be understood in terms of equivalent coupled linear equations defined in a Hilbert space. This was strangely analogous to Quantum Mechanics where also the temporal development is described via a rotation in the Hilbert space. There was obviously some similarity between Quantum Mechanics and Nonlinear Differential equations simply waiting to be exploited. In Quantum Mechanics there is a Hiesenberg frame which is essentially an operator frame work. With this point in mind we looked at the temporal and spatial evolution equations of a particular nonlinear differential equation (Sine Gordon equation) and were able to derive the operator equivalent of the Sine Gordon equation and then solve it to obtain the solution.

In all nonlinear differential equations it is the long wavelength solutions that are important. Both AKNS and ZS have derived the conservation laws corresponding to the KdV equation. However the form of this conservation equation in the long wavelength region has not been considered. We have used the approximation method of Sakaguchi and Malomed to look at the long wavelength behavior of the conservation equation corresponding to the KdV equation. Surprisingly we find Domain wall solutions in this limit. We used the same technique with the Sine Gordon equation. Here again in the long wavelength region we find domain wall solutions. Is this a pattern in the long wavelength region for nonlinear differential equations? At the moment this is not clear.

Next we considered physical systems where domain wall solutions can be found. We find that nonlinear optical lattices admit domain wall solutions. In the case of Josephson junctions kinks and anti- kinks can come together to form a rectangular potential well in which electrons can be trapped. Finally we find that Solitons moving in Poly-acetylene lattices can create vortices

- An important feature of nonlinear differential equations is that they must be solved exactly. In other words perturbation techniques or numerical techniques do not work. In the first paper we published "**Lattice Discretization Approach to Non Linear Differential Equations"** we have taken the Z transform of the difference equation corresponding to the nonlinear differential equation and then solved the recurrence equation exactly. This approach was applied to NLSE, KDV,Sine Gordon equations.
- We find double well in the long wavelength limit in both the KdV and Sine Gordon equations. Since double wells admit TanhSoliton (Domain wall Solitons) solutions we find Tanh solutions in the long wavelength limit in both the KdV and Sine Gordon equations. In the case of the KdV equation the conservation equation is an eigenvalue equation. We have examined this conservation equation in the long wavelength limit using the approximation of Sakaguchi and Malomed. In this approximation we find double well potential and as a result the the conservation equation yields tanhSoliton solutions. In the case of the Sine Gordon equation the same Sakaguchi Malomed approximation leads to a Bessel function expansion. After comparing the coefficients of Cosine term one obtains the Sine Gordon equation in the long wavelength limit. This equation is the same as the equation for the double well and hence we obtain tanhSoliton solution for the Sine Gordon equation in the long wavelength limit. Further in the case of the nonlinear optical lattice we derived from the Hamiltonian used by Sakaguchi and Malomed the Hamiltonian of the double well. Our conjecture was verified as Domain wall Solitons were observed in nonlinear optical lattices. Further in the case of fluid flow we conjectured that the barrier between two fluids is a double well. Here again our prediction of a Domain wall solution is borne out via free energy simulations.
- Ablowitz, Kaup, Newell and Segur (AKNS) and also Zhakarov and Shabat (ZS) have shown that both temporal and spatial evolution equations are associated with a nonlinear differential equation. These evolution equations are linear Eigenvalue equations. Here we used the operator form of the nonlinear differential equations for the Sine Gordon equation and obtained the elliptic equation corresponding to the Sine Gordon equation. On solving the elliptic equation we obtained the tanhsoliton solution.

Abstract of Chapter 1. A general introduction about the soliton and it discovery by John Scott Russell is given. The discovery of soliton challenged was the mathematicians of the time because wave equations known at that time did not admit Soliton solutions. In 1895 Kortweg de Vries, a Dutch mathematician derived the equation of propagation of waves in a shallow water canal. These equations were nonlinear which came to be known as KdV equations, which admit Soliton solutions.

The motivation for this work came from our efforts to understand the Soliton and its interactions. H. Sakaguchi and B.A Malomed in their treatment of the Gross-Pitaevskii equation had developed a very interesting approach to obtain the long wavelength solution. Solitons are known to exist in the DNA. We identify Solitons with jumping genes and suggest that they are responsible for onset of cancer.(Paper presented "Role of Solitons in Jumping Genes and Onset of Cancer" at ICAMTCS 2013, Nagarcoil, Tamil Nadu, ISBN 978-93-82338-30-7)

Abstract of Chapter 2. Review of lituerature has been given in this chapter.

Abstract of Chapter 3. Discretized nonlinear differential equations (KdV and nonlinear Schrodinger equation) have been solved by many authors by multi scale expansions. In this paper we have solved nonlinear differential equations via lattice discretization. This is done by first removing the time dependence of the differential equation and then invoking lattice discretization. This results in a difference equation. For nonlinear equations, lattice discretization results in a nonlinear difference equation which must be solved via other methods. A matlab program has been written which solves the difference equation and plots the solutions from the recurrence relation itself. Via this technique we have solved Nonlinear Schrodinger equation, Toda Lattice equation of motion and Klein-Gordon equation. In each of these cases we have obtained Soliton solutions.The paper titled "Lattice Discretization Approach To Nonlinear Differential Equations" has been published in Global Journal of Pure and Applied Mathematics. ISSN 0973-1768 Volume 10, Number 2 (2014), pp. 225-2314. Available at:http://www.ripublication.com/Volume/gjpamv10n2.htm)

Abstract of Chapter 4. We have used Becker's field theoretic formulation of Navier Stokes equation to derive the equation of motion for a double well potential. We find Domain wall solutions. Double well profiles are found in Cahn-HillardNavier Stokes system. We suggest that Domain walls exist in the Cahn-Hillard system. To investigate this further we compute the Free Energy of the Domain Walls. The plot of the Domain Wall Free Energy agrees very well the profiles computed by other authors. Finally we use the method of Sakaguchi and Malomed to obtain the long wavelength expansion of the Navier Stokes equation. In this limit we find conservation laws. This result implies that under certain conditions the infinite array of conservation laws is also applicable to the Navier Stokes equation.(The paper titled "Long wave length soliton solutions of navier stokes equation" has been published in International Journal of Difference Equations ISSN 0973-6069 Volume 9, Number 1 (2014), pp.1-5Available at: http://www.ripublication.com/ijde/ijdev9n1_01.pdf)

Abstract of Chapter 5. We have extended the technique of Sakaguchi and Malomed to obtain the long wavelength Soliton solutions of the KdV equation. In the long wavelength limit the effective potential is a double well which admits both tanhSoliton solutions as well sinusoidal solutions. (Presented the paper " Long Wave Length Soliton Solutions Of Kdv Equation" at the international Conference ICMMCS IIT Chennai from Dec 08 -10 and published in Scopus Indexed Journal "International journal of Applied Engineering Research(IJAER)" Print ISSN-0973-4562 Online ISSN-1087-1090 (Impact Value SNIP 0.166 (2014) Available at : http://www.ripublication.com/ijaer.htm)

Abstract of Chapter 6. Recently H. Sakaguchi and B.A Malomed proposed a novel technique for finding the long wavelength solutions of the Gross Pitaevskii equation. We have applied the technique of Sakaguchi and Malomed to the Sine Gordon equation and derived the equivalent conservation equation. The results are applied to the Josephson junction. Papertitiled **" Long Wave Length TanhSoliton Solutions Of Sine Gordon Equation" has been** presented at International Conference at Jamia Millia Islamia New Delhi, on Algebra, Geometry, Analysis and their Applications (ICAGAA-14) Nov 27-29, 2014 and published in the International journal of Engineering and Technical researchs. ISSN 2321-0896,Vol -3 Issue-6 July 2015 (Impact factor 1.315 in 2014)https://www.erpublication.org/IJETR/index.php

Abstract of Chapter7. From the Gross-Pitaevskii equation for nonlinear optical lattice we derive the Hamiltonian for the double well model. This model is solved to obtain tanh domain wall soliton solutions which have been observed and also derived by other authors using a different model. The domain wall soliton solutions predict lattice compressibility which has been observed. Further, from this model we obtain and simulate the probability for tunneling from one well to another which agrees with experimental results.(Paper titled "**Domain Wall Soliton Solution In Nonlinear Optical Lattices"** has been Presented in the WASET International Conference, Bali Indonesia, Oct. 26-28 2011 and published. pISSN 2010-376X eISSN 2010-378)Available at:http://waset.org/Publication/solitons-in-nonlinear-optical-lattices/5260

Abstract of Chapter 8. We develop a model to account for the recently observed phase jump of electrons in Josephson junction, in a magnetic field, as the electrons cross the junction. We suggest that electrons are trapped in the potential formed by a kink anti-kink pair. When the electron escapes from this potential well it suffers a potential jump as it crosses the junction. Electrons at lower depths suffer greater potential jumps. The potential jumps were evaluated by using the Lax pair for the Sine Gordon equation and then using Gelfand-Levitan equation on the bound states formed by the kink-anti kink pair.**(Paper titled "Long Josephson Junctions In Magnetic Field"** has been **published** in the International journal of Engineering and Applied Sciences. ISSN 2394-3661,Vol -2 Issue-5 may 2015 (Impact factor 1.22 in 2014

Available at: https://www.ijeas.org/download_data/IJEAS0205050.pdf)

Abstract of Chapter 9.Solitons in Poly-Acetylene have been both predicted theoretically and found experimentally. However as is shown in this paper Vortex Soliton solutions are also possible. One feature of the Vortex solutions is that a small change in the parameters (bond angles) induces a large change in the interaction potential. This induces extended discontinuities in the wave vector space of the system. A large block of states are even absent due to the presence of vortices. A depleted number of states is thus forced to accommodate the original number of spins and charges in the system. The only way this is possible that two spins pair up to form a spin less charged Vortex. As a result vortices may be with spin or without spin and also with or without charge. This phenomenon is confirmed via numerical simulations on Poly-Acetylene.(Paper titled "Vortex SolitonsIn Poly-Acetylene" has been presented at the International conference, Union of Pure and Applied Physics- IUPAP at IIT Guwahati, Dec 5-7, 2015 and published in International Journal of Science and Research (IJSR) ISSN (online) no 2319-7064 Index Copernicus Value (2013) 6.14 Impact factor (2015) :6.391Available at:https://www.ijsr.net/archive/v5i7/v5i7_01.php)

Abstract of Chapter 10. Ablowitz, Kaup, Newell and Segur (AKNS) and also Zhakarov and Shabat (ZS) have shown that both temporal and spatial evolution equations are associated with a nonlinear differential equation. These evolution equations are linear Eigenvalue equations. For the unperturbed Sine-Gordon equation, the spatial evolution equation may be interpreted as a rotation in the potential space. And temporal evolution is the same as a rotation matrix in potential space through an angle u. We solve for the Eigen values for this rotation operator. The Eigen values, which are in the form of operators, are solved. Via this technique we solve the perturbed Sine-Gordon equation. In the small amplitude limit we recover the kink solution of the Sine-Gordon equation implying that the operator approach employed here is correct. This result implies that evolution of all nonlinear differential equation can be thought of as a transformation of the potential space.(Paper titled**"Soliton Solution Of The Unperturbed Sine-Gordon And Perturbed Sine-Gordon**

Equation" has been presented at the International Conference on Applied and Mathematical Models ICAMM 2016 Coimbatore Jan 5-7, 2016 and published in Scopus Indexed Journal "International journal of Applied Engineering Research(IJAER)" Print ISSN-0973-4562 Online ISSN-1087-1090 (Impact Value SNIP 0.166 (2014) Available at http://www.ripublication.com/ijaer.htm

Abstract of Chapter 11. ConclusionAnd Future Work: The diverse topics treated in this thesis can be attributed to the many different areas nonlinear dynamics is applicable. This is an ever growing field very far from its saturation. Therefore more and more applications of this area are likely to come. However we outline a few areas where we would like to continue to work and develop.

Our work started with the observation that the Lagrangian used by Sakaguchi and Malomed correspond to that of the double well. Since double wells admit domain wall solutions it was obvious that nonlinear lattices would also admit domain wall soliton solutions. One of the points we have made is that the particular case Sine-Gordon equation can be treated as a rotation in the potential space. Our point of view is that every nonlinear differential equation can be treated as some transformation of the potential space. We would like to prove this conjecture with other nonlinear differential equations.

We have extended the Inverse scattering framework to the stochastic domain in our paper entitled probabilistic inverse scattering. In inverse scattering the temporal and spatial evolution of the state vector is given by rotations in the Hilbert space. In the case of the probabilistic system (stochastic system) the time evolution is given by random rotations in the Hilbert space.

We found Harmonic Oscillator type of states in the solutions of number nonlinear differential equations (KdV, Phi4, Sine-Gordon, Navier Stokes). It is of interest to know whether such bound states can be used for information transfer.

FUTURE WORK

We would like to study how nonlinear dynamics affects the electronic structure of nonlinear materials. This is particularly evident in rare earth High Temperature Super Conductivity compounds such as $YBa_2Cu_3O_7$. In such compounds a pseudo gap appears near the Fermi surface. While the origin of this pseudo gap is still in doubt it is our conjecture that the onset of Solitons caused by the phase transformation produces this pseudo gap. It is well known that RVB states or Quantum Fluids form magnetic domain walls. Earlier we have studied domain wall solutions in Navier Stokes equations (Chapter 4). Thus we can interpret High Tc as the onset of Magnet Domains in Quantum Fluids.

In nonlinear optical crystals such domain walls can act as modulators. That is domain walls can induce mid gap states which in turn can absorb energy. Now the effect of domain walls on the electronic energy band gap has been worked out by Su, Schrieffer and Heeger (SSH) in their seminal paper. However a domain wall produces a deformation of the lattice which can be expressed in terms of the conduction band and valence band states. This was done by SSH. However the equation they have derived is the ZS equation. Hence one can write the Gelfand-Levitan equation corresponding to the above system. This allows us describe Soliton states in the lattice in the presence of mid gap states. In the Krumhansl and Schrieffer paper we had simply a linear array of atoms interacting via double well potential. In the SSH paper we have linear array of atoms interacting via double well potential and having a conduction band as well. The conduction succeeds in modifying both the spin and charge of the Solitons. However our interest lies in sandwich materials made of electro –optic materials such as lithium niobate. Via external modulation we want to create a state in the visible photon absorption range. Absorbed photons in this range should de-excite in the form of Solitons in Lithium Niobate from where it can be extracted in the form of current. This technique allows us to develop a fascinating energy storing device.