CHAPTER 11

CONCLUSIONS AND FUTURE WORK

11.1 CONCLUSIONS

The Aims and Objectives (1.2) were achieved and published the results.

1.2.1 We find numerical soliton solution of Nonlinear Schrodinger equation.

Paper Published: “Lattice Discretization Approach to Nonlinear Differential Equations”, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768, Volume 10, Number 2, (2014) pp.225-231

Available at: **http://www.ripublication.com/Volume/gjpamv10n2.htm**

Description:

- The paper on lattice discretization was made possible by converting the nonlinear differential equation into an equivalent difference equation and then solving the difference equation exactly in Matlab. For each nonlinear differential equation we obtained Soliton profile.

1.2.2 Long Wavelength Soliton Solution of Navier Stokes Equation

Paper Published: “Long Wavelength Solitons Solution of Navier Stokes Equation”, International Journal of Difference Equations, ISSN Print ISSN 0973-6069
ISSN 0973-6069 Volume 9, Number 1 (2014), pp. 1-5
Available at: http://www.ripublication.com/ijde/ijdev9n1_01.pdf

Description:

- We noted that the Free Energy profile of the two fluid models is given by a double well allowing us to use the formalism of Krumhansl and Schrieffer.

1.2.3 We got Solitons Solution of the KdV equation in the long wavelength limit

Paper Published: “Long wave length Tanh soliton solution of KdV equation”,
International Journal of Applied Engineering Research, Print ISSN 0973-4562
Online ISSN 0973-9769
Availabe at: http://www.ripublication.com/Volume/ijaerv11n7.htm

Description:

- Here we extend the technique of Sakaguchi and Malomed of finding long wavelength solutions to the KdV equation. We find that bound states are propagated with the Solitons

1.2.4 We find Soliton Solution of the Sine Gordon equation

Paper Published: "Long Wave Length tanh Soliton Solution of Sine Gordon Equation", ISSN : 2321-086

Available at:
https://www.erpublication.org/admin/vol_issue1/upload%20Image/IJETR032547.pdf

Description:

- Here equation is solved with the approximation proposed by Sakaguchi and Malomed. The results are applied to The Josephson's junction.

1.2.5 Solution of the Sine Gordon equation (unperturbed and perturbed) via the method of D.J Kaup and El-sayed Osman [73-74] and AKNS [6].

Paper Published: **"Solution of the unperturbed Sine Gordon equation and Perturbed Sine-Gordon Equation"**, Print ISSN: 0973-4562, Online ISSN: 0973-9769

Available at: http://www.ripublication.com/Volume/ijaerv11n1spl.htm

Description:

- For the unperturbed Sine Gordon equation the spatial evolution equation may be treated as a rotation in potential space. A Soliton solution is thus equivalent to a rotation through a large angle. We use the operator approach to solve the equations.

1.2.6 We find Soliton solution in nonlinear optical lattices.

Paper Published: "Soliton in Nonlinear optical lattices", pISSN: 2010-376X, eISSN: 2010-3778

Available at:http://waset.org/Publication/solitons-in-nonlinear-optical-lattices/5260

Description:

- Our work started with the observation that the Lagrangian used by Sakaguchi and Malomed correspond to that of the double well. Since double wells admit domain wall solutions it was obvious that nonlinear lattices would also admit domain wall soliton solutions.

1.2.7 Soliton solutions in Long Josephson junctions in a magnetic Field

Paper Published "Long Josephsons Junction in Magnetic Field", ISSN : 2394-3661,

Available at: https://www.ijeas.org/download_data/IJEAS0205050.pdf

Description:

- Here we examine the conjecture of a bound state formed by a kink and anti-kink. Electrons get trapped in the potential well formed by the kink and anti-kink pair. Electrons escaping from these bound states have a definite phase difference which has been observed.

1.2.8. Vortex Soliton solution in Poly-acetylene.

Paper Published: "Vortex Soliton in Poly-acetylene", International Journal of Science and Research (IJSR), ISSN: 2319-7064, Volume 5 Issue 7, July 2016 pp 1092-1096

Available at:https://www.ijsr.net/archive/v5i7/v5i7_01.php

Description:

- It was recognized that a domain wall separates regions with different types of bonding (cis and trans). Here we use the techniques used for nonlinear optical lattices for BEC condensates to describe a Poly acetylene lattice and obtained vortex excitations.

The diverse topics treated in this thesis can be attributed to the many different areas nonlinear dynamics is applicable. This is an ever growing field very far from its saturation. Therefore more and more applications of this area are likely to come. However we outline a few areas where we would like to continue to work and develop.

Our work started with the observation that the Lagrangian used by Sakaguchi and Malomed correspond to that of the double well. Since double wells admit domain wall solutions it was obvious that nonlinear lattices would also admit domain wall soliton solutions. One of the points we have made is that the particular case Sine-Gordon equation can be treated as a rotation in the potential space. Our point of view is that every nonlinear differential equation can be treated as some transformation of the potential space.

In the case of Josephson junctions in a magnetic field we examine the conjecture of a bound state formed by a kink and anti kink. The paper on lattice discretization was made possible by converting the non linear differential equation into and equivalent difference equation and then solving the difference equation exactly in Matlab. Similarly we noted that the Free Energy profile of the two fluid model is given by a double well allowing us to use the formalism of Krumhansl and Schrieffer. We further found that shallow water bodies are described by the Kdv equation. In the low amplitude limit this equation has sinusoidal solutions while in the large amplitude limit this has tanh soliton solutions. We next looked at Poly Acetylene. It was recognized that a domain wall separates regions with different types of bonding.Here we use the techniques used for non linear optical lattices for BEC condensates to describe a Poly acetylene lattice and obtained vortex excitations. We have already interpreted the AKNS transformation as a transformation in the potential space. Next we look at the perturbation in this potential space for the Sine Gordon equation.

We have extended the Inverse scattering framework to the stochastic domain in our paper entitled probabilistic inverse scattering. In inverse scattering the temporal and spatial evolution of the state vector is given by rotations in the Hilbert space. In the case of the probabilistic system (stochastic system) the time evolution is given by random rotations in the Hilbert space.

We found Harmonic Oscillator type of states in the solutions of number nonlinear differential equations (KdV, Sine-Gordon, and Navier Stokes). It is of interest to know whether such bound states can be used for information transfer.

11.2 FUTURE WORK

We would like to study how nonlinear dynamics affects the electronic structure of nonlinear materials. This is particularly evident in rare earth High Temperature Super Conductivity compounds such as $YBa_2Cu_3O_7$. In such compounds a pseudo gap appears near the Fermi surface. While the origin of this pseudo gap is still in doubt it is our conjecture that the onset of Solitons caused by the phase transformation produces this pseudo gap. It is well known that RVB states or Quantum Fluids form magnetic domain walls. Earlier we have studied domain wall solutions in Navier Stokes equations (Chapter 9). Thus we can interpret High Tc as the onset of Magnet Domains in Quantum Fluids.

In nonlinear optical crystals such domain walls can act as modulators. That is domain walls can induce mid gap states which in turn can absorb energy. Now the effect of domain walls on the electronic energy band gap has been worked out by Su, Schrieffer and Heeger (SSH) in their seminal paper. However a domain wall produces a deformation of the lattice which can be expressed in terms of the conduction band and valence band states. This was done by SSH. However the equation they have derived is the ZS equation. Hence one can write the Gelfand-Levitan equation corresponding to the above system. This allows us describe Soliton states in the lattice in the presence of mid gap states. In the Krumhansl and Schrieffer paper we had simply a linear array of atoms interacting via double well potential. In the SSH paper we have linear array of atoms interacting via double well potential and having a conduction band as well. The conduction succeeds in modifying both the spin and charge of the Solitons. However our interest lies in sandwich materials made of electro –optic materials such as lithium niobate. Via external modulation we want to create a state in the visible photon absorption range. Absorbed photons in this range should de-excite in the form of Solitons in Lithium Niobate from where it can be extracted in the form of current. This technique allows us to develop a fascinating energy storing device.