**Write the following information in the first page of Answer Script before starting answer**

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number______________________________

Course____________________________ Semester____________________________

Paper Code _________________Paper Title __________________________________

Type of Exam: ______________________________ (Regular/Back/Improvement)

**Important Instruction for students:**

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

# M.Sc. MATHEMATICS
## THIRD SEMESTER
## MATHEMATICAL METHODS
## MSM-303

**Duration : 3 hrs.** **Full Marks : 70**

**( PART-A : Objective )**

**Time : 20 min.** **Marks : 20**

***Choose the correct answer from the following:*** ***1X20=20***

**1.** $L^{-1}\left\{\dfrac{1}{s^{n+1}}\right\}$ is:

**a.** $\dfrac{t^{n+1}}{n\,!}$ **b.** $\dfrac{t^{n-1}}{n\,!}$

**c.** $\dfrac{t^{n-1}}{(n+1)\,!}$ **d.** None

**2.** The values of $L\{t^{n+1}\}$ is:

**a.** $\dfrac{n!}{S^n}$ **b.** $\dfrac{(n+1)}{S^{n+1}}$

**c.** $\dfrac{t^{n-1}}{n\,!}$ **d.** None

**3.** The value of $L\{-1\}$ is:

**a.** 0 **b.** -1

**c.** $-\dfrac{1}{S}$ **d.** None

**4.** Boundary value problems in the theory of ordinary differential equation can lead to integral equations of the type:

**a.** Volterra **b.** Fredholm

**c.** Mellin **d.** Laplace

**5.** If the upper limit of the integral equation is not a constant then the equation is of the type:

**a.** Volterra **b.** Fredholm

**c.** Hankel **d.** Holbert

**6.** $L\left(e^{at}t^n\right)$ is:

**a.** $\dfrac{n!}{(S-a)^{n+1}}$ **b.** $\dfrac{n!}{(S-a)^n}$

**c.** Both a and b **d.** None of the above

7. Linear integral equation of the form,

$$\phi(x) = f(x) + \lambda \int_a^b k(x,\xi)\,\phi(\xi)\,d\xi$$ is known as Fredholm integral equation of:

**a.** 1st kind
**b.** 2nd kind
**c.** 3rd kind
**d.** None

8. A linear integral equation of the form,

$$y(x) = \lambda \int_a^b k(x,t)\,y(t)\,dt$$ is called homogeneous Volterra integral equation of:

**a.** 1st kind
**b.** 2nd kind
**c.** 3rd kind
**d.** All of the above

9. Formula to convert multiple integral

$$\int_a^x y(t)\,dt^n$$ into a single ordinary integral is:

**a.** $\int_a^x \frac{(x-t)^n}{n!} y(t)\,dt$
**b.** $\int_a^x \frac{(x-t)^{n-1}}{(n-1)!} dt$
**c.** $\int_a^x \frac{(x-t)^n}{n!} dt$
**d.** None

10. Find $L\left(t^{1/2}\right)$

**a.** $\frac{\sqrt{\pi}}{S^{3/2}}$
**b.** $\frac{\sqrt{\pi}}{4S^{5/2}}$
**c.** $S^{3/2}$
**d.** None

11. $L(F(t)) = f(S)$ then $L\left(e^{at}F(t)\right) = f(S-a)$ is called:

**a.** 1st shifting theorem
**b.** 2nd shifting theorem
**c.** Both a and b
**d.** None

12. Inverse Laplace transform of $\frac{1}{\sqrt{S}}$ is:

**a.** $\frac{t^{1/2-1}}{\Gamma(1/2)}$
**b.** $\frac{t^{1/2}}{\Gamma(1/2)}$
**c.** Both a and b
**d.** None

**13.** Fourier transform is defined on:

**a.** $(-\infty, \infty)$ **b.** $(-\infty, 0)$

**c.** $(0, \infty)$ **d.** $[0, \infty)$

**14.** Which of the following can't be a kernel of cosine transformation?

**a.** $Sin\ sx$ **b.** $e^{-isx}$

**c.** $e^{sx}$ **d.** All of the above

**15.** If $F(S)$ is the Fourier transformation of $F(x)$ then Fourier transformation of $F(kx)$ is:

**a.** $\frac{1}{k}F\left(\frac{S}{k}\right)$ **b.** $F\left(\frac{S}{k}\right)$

**c.** $F(ks)$ **d.** $F(sx)$

**16.** $L(0)$:

**a.** 0 **b.** 1

**c.** Undefined **d.** None

**17.** Which can't be the eigen value of the equation,

$$y(x) = \lambda \int_a^b k(x,t)\, y(t)\, dt$$

**a.** $\lambda = 0$ **b.** $\lambda = 1$

**c.** $\lambda = 2$ **d.** None

**18.** $L(\frac{1}{(s-2)^2}) = ..$

**a.** $e^t$ **b.** $te^t$

**c.** $e^{2t}$ **d.** None

**19.** If $L^{-1}(\frac{a}{(s+b)^2 - a^2}) =,$

**a.** $e^{bt} Sinhat$ **b.** $e^{-bt} Sinhat$

**c.** $e^{bt} Sinhbt$ **d.** None

**20.** $L^{-1}\left(\frac{1}{S^{n+1}}\right) = \frac{t^n}{\Gamma(n+1)}$ then:

**a.** $n \geq -1$ **b.** $n > -1$

**c.** n is rational **d.** n is positive rational

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## ( PART-B : Descriptive )

**Time : 2 hrs. 40 min.** **Marks : 50**

**[ Answer question no.1 & any four (4) from the rest ]**

**1.** **a.** Form an integral equation corresponding to the differential equation, 6+4=10

$$y''' - 2xy = 0$$

with initial conditions, $y(0) = \frac{1}{2}, y'(0) = y''(0) = 1.$

**b.** Find the eigen values and corresponding eigen function of the integral equation

$$y(x) = \lambda \int_0^1 (6x - t) y(t) dt$$

**2.** **a.** Show that the linear differential equation of 2nd order 6+4=10

$$\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = F(x)$$

with initial conditions $y(0) = c_0$, $y'(0) = c_1$ can be transformed into non-homogeneous Volterra equation of 2nd kind.

**b.** Find $L^{-1}\left\{\frac{S}{(S^2-1)^2}\right\}$

**3.** **a.** Find Fourier transformation of $F(x)$ defined by, 6+4=10

$$F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

And hence evaluate,

(i) $\int_{-\infty}^{\infty} \frac{Sinax\, CosSx}{S} dx$

(ii) $\int_0^{\infty} \frac{SinS}{S} dS$

**b.** Apply convolution theorem to find,

$$L^{-1}\left\{\frac{S^2}{(S^2+a^2)(S^2+b^2)}\right\}$$

**4.** **a.** Find Fourier Sine and Cosine transformation of $f(x)$ if, 6+4=10

$$f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$$

**b.** $L^{-1}\left\{\dfrac{S}{S(S+1)^3}\right\}$

**5.** **a.** Show that $y(x)=\cos 2x$ is a solution of the integral equation 6+4=10

$$V(x)=\cos x+3\int_0^{\pi} k(x,t)y(t)dt$$

Where $k(x,t)=\begin{cases} \sin x\cos t & 0\le x\le t \\ \cos x\sin t & t\le x\le \pi \end{cases}$

**b.** Evaluate $L\left\{t^2e^{2t}Sin3t\right\}$

**6.** **a.** Using Laplace transformation solve the following differential equation, 6+4=10

$$\frac{d^3x}{dt^3}-3\frac{d^2x}{dt^2}+3\frac{dx}{dt}-x=t^2e^t, x(0)=1, x'(0)=0$$

**b.** Find Laplace transformation of:

**(i)** $Coshat\ Sinhat$

**(ii)** $Cos^3 3t$

**7.** **a.** What is the integral equation of Convolution type? 1+1+1+1+6=10

**b.** What is the Leibnit'z rule of differentiation under integral sign?

**c.** What is the homogeneous integral equation of 2nd kind?

**d.** Write Volterra equation of 2nd kind.

**e.** Write a note on Mellin and Hankel transformation.

**8.** **a.** Transform the boundary value problem 6+4=10

$$\frac{d^2y}{dx^2}+y=x, \quad y(0)=0,\ y'(1)=0$$

To Fradholm integral equation

$$y(x)=\frac{1}{6}(x^3-3x)+\int_0^1 K(x,t)y(t)dt \text{ where}$$

$$y(x)=\begin{cases} x & , x<1 \\ t & , x>1 \end{cases}$$

**b.** Solve the following homogeneous integral equation:

$$y(x) = \frac{1}{e^2 - 1}\int_0^1 2e^x e^t y(t)\,dt$$

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