Write the following information in the first page of Answer Script before starting answer

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number_____

Course_____   Semester_____

Paper Code _____Paper Title _____

Type of Exam: _____   (Regular/Back/Improvement)

Important Instruction for students:

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

B.Sc. PHYSICS/B.Sc. CHEMISTRY
THIRD SEMESTER
CLASSICAL ALGEBRA & TRIGONOMETRY
BSM-731

Duration : 3 hrs.   **Full Marks : 70**

(**PART-A : Objective**)
Time : 20 min.   **Marks : 20**

Choose the correct answer from the following: *1X20=20*

1. Which inequality is known as Cauchy-Schwartz's inequality?
   a. $\left(\sum_{i=1}^{n} a_i b_i\right) \geq \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i\right)$ **b.** $\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$

   c. $\left(\sum_{i=1}^{n} a_i^2 b_i^2\right) \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$ **d.** None

2. The condition $AM = GM$ holds when the quantities are:
   a. Equal   **b.** Unequal
   c. Hold for any number   **d.** None

3. A system of equations is said to be consistent and it has unique solution when, determinant of the matrix is:
   a. Non-zero   **b.** Less than zero
   c. Zero   **d.** Greater than zero

4. In which case an inconsistent system of equations has no solution?
   a. (adj A).B≠0   **b.** (adj A).B=0
   c. (adj A). B<0   **d.** None

5. Which is the correct expression for $\frac{(\cos\theta - i\sin\theta)^{10}}{(\cos\theta + i\sin\theta)^{11}}$
   a. $\cos 22\theta - i\sin 22\theta$ **b.** $\cos 22\theta + i\sin 22\theta$

   c. $\cos 22\theta - i\sin 2\theta$ **d.** None

6. Every equation of nth degree has:
   a. n-1 roots   **b.** n+1 roots
   c. n roots   **d.** None

7. If $x = -3 + 2i, y = -3 - 2i, then\ x^2 + y^2 + xy =?$
   a. 40   **b.** 23
   c. 41   **d.** 43

8. Find the conjugate of the complex number -2-3i
   a. 2+3i   **b.** -2+3i
   c. 2-3i   **d.** None

9. The value of $\sqrt{i} + \sqrt{-i}$=?

    a. 2   **b.** 0

    c. $\sqrt{-2}$ **d.** $\sqrt{2}$

10. If z is purely imaginary, then $z + \bar{z}$ =?

    a. z   **b.** 0

    c. -z   **d.** None

11. (1-i) can also be expressed in the form:

    a. $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ **b.** $\sqrt{3}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

    c. $\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$ **d.** None

12. A system of equation is said to be inconsistent when it has:

    a. One or more solution   **b.** No solution

    c. Both (a) and (b)   **d.** None

13. If α, β, γ are roots of the equation $x^3 + qx + r = 0$ then what is the value of $\sum \alpha^2 \beta$

    a. -3r   **b.** 3r

    c. 2r   **d.** None

14. If z is a non-zero complex number, then $z\bar{z}$ is:

    a. Purely real   **b.** Purely imaginary

    c. Zero   **d.** None

15. Find the determinant of $\begin{bmatrix} 23 & 33 & 44 \\ 23 & 0 & -666 \\ 23 & 33 & 44 \end{bmatrix}$

    a. 1   **b.** 2

    c. 0   **d.** None

16. Find the co-factor of 3 in the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 5 & 1 \\ 3 & 7 & 1 \end{bmatrix}$

    a. -1   **b.** 2

    c. 1   **d.** None

17. For positive integers a, b, c and d then what is the value of $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}$ is:

    a. Greater than 4   **b.** Equal to 4

    c. Less than 4   **d.** None

18. Find the correct statement?

    a. Matrix multiplication is commutative   **b.** Matrix multiplication is not distributive

    c. Matrix multiplication is not   **d.** None
       commutative

19. Which property is not true in case of matrix?

    a. $A + B = B + A$ **b.** $K(A + B) = KA + KB$

    c. AB=BA   **d.** A(B+C)=AB+AC

20. Find x, y, z and t which satisfy the matrix equation $\begin{bmatrix} x - y & 2x + z \\ 2x - y & 2z + t \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

    a. $x = 1, y = 2, z = 3, t = -1$ **b.** $x = 1, y = -2, z = 3, t = -1$

    c. $x = 1, y = 2, z = 3, t = 7$ **d.** None

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(PART-B : Descriptive)

Time : 2 hrs. 40 min.   **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. State Cauchy-Schwartz inequality with example. If a,b,c are positive real numbers then prove that   3+7=10
$$a^2(b+c) + b^2(c+a) + c^2(a+b) \geq 6abc.$$

2. **a.** For any three positive real numbers a, b and c prove that   4+6=10
$$\frac{a^2+b^2}{a+b} + \frac{b^2+c^2}{b+c} + \frac{c^2+a^2}{c+a} \geq a+b+c$$
b. What is the logarithmic expansion of $(1+z)$. Evaluate $\log(\alpha + \quad i\beta)$ where α and β are real.

3. **a.** Solve the following equation $x^7 + x^4 + x^3 + 1 = 0$.   4+6=10
b. State and prove De Moivre's Theorem

4. **a.** Let a, b, c denote the sides of a triangle. Prove that   6+4=10
$abc \geq 8(S- \quad a)(S-b)(S-c)$, where $2S = a+b+c$
b. If $a^2 + b^2 + c^2 = 1$ then show that $-\frac{1}{2} \leq ab+bc+ca \leq 1$

5. If z_1 and z_2 be two complex numbers then prove that   5+5=10
a. $|z_1 z_2| = |z_1||z_2|$ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$.
b. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ and $arg\left(\frac{z_1}{z_2}\right) = arg z_1 - arg z_2$.

6. **a.** If z_1 and z_2 are two complex numbers such that $\frac{z_1}{z_2}$ is purely   4+3+3=10
imaginary, prove that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$
b. (i) Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew symmetric matrix.
(ii) Solve the system of equation by matrix method
$$5x + 7y + 2 = 0, \quad 4x + 6y + 3 = 0.$$

7. **a.** If α, β are roots of $x^2 - 2x + 4 = 0$ then prove that   5+5=10
$$\alpha^n + \beta^n = 2^{n+1} \cos\frac{n\pi}{3}.$$

b. Find the inverse of the following matrix A= $\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$

8. **a.** If z is a complex number and $\frac{z-1}{z+i}$ is purely imaginary, show that the   4+4+2=10
point z lies on the circle whose centre is at the origin and whose radius is 1.
b. (i) If
z_1 and z_2 be two complex numbers then prove that $|z_1 + z_2| \leq \quad |z_1| + |z_2|$.
(ii) What is the centre and radius of the equation of circle
$$x^2 + y^2 + \quad x - y = 0$$
$$= = *** = =$$