M.Sc. PHYSICS
First Semester
CLASSICAL MECHANICS
(MPH - 102)

Duration: 3Hrs. **Full Marks: 70**

Part-A (Objective) =20
Part-B (Descriptive) =50

(PART-B: Descriptive)

Duration: 2 hrs. 40 mins. **Marks: 50**

Answer any *four* from *Question no.* 2 to 8
***Question no.* 1 is compulsory.**

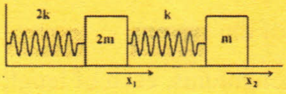
1. (a) Applying variation principle, show that the shortest distance between two points in a plane is a straight line.

   (b) What describe the principle of least action, and then deduce

$$\Delta \int_{t_1}^{t_2} \sum_j p_j \dot{q}_j = 0$$

(5+5=10)

2. Obtain Lagrange's equation of motion from D'Alembert's principle. Applying Lagrange's equation find the normal mode frequencies for the spring mass system given below (5+5=10)



3. Deduce Hamilton's canonical equations of motion and apply them to obtain

   a) Equation of motion for a linear harmonic oscillator,

   b) Equation of motion for a simple pendulum. (5+2+3=10)

4. (a) Prove that the transformation from space set of axes to body set of axes, i.e., $A - BCD$ is orthogonal, where B, $C$and D are the transformation matrices of the first, second and the third rotation respectively.

   (b) Define the following:

   (i) Euler's angle, (ii) Inertia Tensor, (iii) Principal axes

   (c) A rigid body consists of three particles of masses 2, 1, 4 grams located at (1, -1, 1), (2, 0, 2), and (-1, 1, 0) respectively. Determine the inertia tensor.

(2+3+5=10)

5. (a) Give the physical significance of Hamilton's characteristic function w.

   (b) Discuss the one dimensional Harmonic oscillator problem using Hamilton-Jacobi method. (2+8=10)

6. (a) Define Poisson's bracket. If $[\alpha, \beta]$ be the Poisson's bracket, then prove that

$$\frac{\partial}{\partial t}[\alpha, \beta] = \left[\frac{\partial \alpha}{\partial t}, \beta\right] + \left[\alpha, \frac{\partial \beta}{\partial t}\right]$$

   (b) Give one condition for a transformation to be canonical. Show that the following transformation is canonical. (5+5=10)

$$Q = p, \; P = -q$$

7. (a) Two identical pendulums, each of mass m and effective length l are coupled together by a light spring of force constant k. Determine T and V matrices, where T and V are the kinetic and potential energies of the system respectively. Also find the normal frequencies of the system.

   (b) Two identical simple pendulums, each of length 0.4 m, are connected by a light spring. The force constant of the spring is 1 Nm^{-1} and mass of each bob is 50 gm. If one pendulum is clamped, calculate the period of the other pendulum. When the clamp is removed, determine the periods of two normal modes of the system. (g=9.8 ms^{-2})

(5+2+3=10)

8. Obtain the Euler's equations of motion for a rotating rigid body, with a fixed point. From Euler's equations of motion, show that in absence of external torque, both the total rotational kinetic energy and total angular momentum are conserved.

(5+5=10)

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REV-00
MPH/57/62 2016/12

M.Sc. PHYSICS
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Duration: 20 minutes **Marks – 20**

(PART A - Objective Type)

I. Choose the correct answer: **1×20=20**

1. Unilateral constants are expressed as
   (a) equations (b) inequalities
   (c) vectors (d) scalars

2. If the Lagrangian does not depend on time explicitly, then the
   (a) Hamiltonian is constant
   (b) Hamiltonian cannot be constant
   (c) Kinetic energy is constant
   (d) Potential energy is constant

3. If the Lagrangian of a system is not the function of a given co-ordinate q_k, then the co-ordinate is said to be
   (a) non-ignorable (b) cyclic
   (c) generalised (d) none

4. What is the least requirement of coordinates to specify the motion of a rigid body?
   (a) 0 (b) 3 (c) 6 (d) 12

5. In case of inertia tensor, I_{xy} is expressed as.
   (a) $I_{xy} = \sum_i m_i x_i y_i$ (b) $I_{xy} = -\sum_i m_i x_i y_i$
   (c) $I_{xy} = -\sum_i m_i(x_i^2 + y_i^2)$ (d) $I_{xy} = 0$

6. The relation between the instantaneous linear velocity (V) and instantaneous angular velocity (ω) of a point at 'r' relative to an origin 'O' is given by,
   (a) $V = \frac{\omega}{r}$ (b) $V = \omega + r$
   (c) $V = \omega(\omega \times r)$ (d) $V = \omega \times r$

7. A finite rotation or angular displacement
   (a) can be represent by a vector (b) cannot be represent by a vector
   (c) can be represent by a scalar (d) none of these

8. The precessional period of a symmetrical rigid body about the symmetry axis is
   (a) $T - \frac{2\pi}{\Omega}$ (b) $T - \frac{2\pi}{\omega}$ (c) $T - \frac{\omega}{2\pi}$ (d) $T = \frac{I_1}{I_3 - I_1}$

9. Property of an ideal fluid is/are
   (a) incompressible (b) laminar flow
   (c) non-viscous (d) all of them

10. Bernoulli's equation is a statement of

(a) energy conservation
(b) hydrostatic equilibrium
(c) compressibility of liquid
(d) none of these

11. Hamilton's principle function S and Hamilton's characteristic function W for a conservative system are related as

(a) $S = W$
(b) $S = W - Et$
(c) $S = W + Et$
(d) S is not related to W

12. For an one dimensional harmonic oscillator, the representative point in two dimensional phase space traces

(a) an ellipse
(b) a parabola
(c) a hyperbolae
(d) always a straight line

13. The action and angle variables have the dimensions of

(a) force and angle
(b) angular momentum and angle
(c) energy and angle
(d) are dimensionless quantities

14. If the generating function has the form $F = F(q_k, Q_k, t)$ then the transformation equations are

(a) $p_k = \frac{\partial F}{\partial q_k}, P_k = -\frac{\partial F}{\partial Q_k}$
(b) $p_k = -\frac{\partial F}{\partial q_k}, P_k = \frac{\partial F}{\partial Q_k}$
(c) $p_k = \frac{\partial F}{\partial Q_k}, P_k = -\frac{\partial F}{\partial q_k}$
(d) $p_k = -\frac{\partial F}{\partial q_k}, P_k = -\frac{\partial F}{\partial Q_k}$

15. In case of two coupled identical pendulums oscillating in a plane

(a) each pendulum always executes simple harmonic motion.
(b) two pendulums may execute simple harmonic motion.
(c) the general motion can be expressed as a superposition of two simple harmonic motions of same frequency.
(d) the general motion can be expressed as a superposition of two simple harmonic motions of different frequency.

16. In case of a linear triatomic molecule XY_2 type, the eigen frequencies ω_1, ω_2 and ω_3 can be represented as

(a) $\omega_1 = \omega_2 = \omega_3$
(b) $\omega_1 = 0,\ \omega_2 = \omega_3$
(c) $\omega_1 = 0,\ \omega_2 \neq \omega_3$
(d) $\omega_1 = 1,\ \omega_2 = \omega_3$

17. If the Poisson bracket of a function with the Hamiltonian vanishes, then

(a) the function depends upon time.
(b) the function is a constant of motion.
(c) the function's characteristic depends on the Hamiltonian.
(d) the Hamiltonian is zero.

18. The correct relations for fundamental Poisson's bracket are

(a) $[q_k, q_l] = \delta_{kl}$
(b) $[q_k, q_l] = 0$
(c) $[q_k, p_k] = 0$
(d) $[q_k, p_k] = 1$

19. The phase space refers to
   (a) position coordinate.
   (b) momentum coordinate.
   (c) both position and momentum coordinates.
   (d) none of the above.

20. Saddle point
   (a) is stable.
   (b) is unstable.
   (c) corresponds to oscillatory motion.
   (d) corresponds to damped oscillatory motion.

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