7. a. Show that any inner product of the tensors A^p and B_q is a scalar. 3+3+4=10

b. Prove that $A_{\mu\nu}B^{\mu}C^{\nu}$ is an invariant if B^{μ} and C^{ν} are contravariant tensors of rank 1 and $A_{\mu\nu}$ is a covariant tensor of rank 2.

c. Show that if a tensor is symmetric with respect to two indices in any coordinate system, it remains symmetric with respect to these two indices in any other coordinate system.

8. a. Show that: 2+3+5=10

$[\mu\nu, \sigma] + [\sigma\nu, \mu] = \frac{\partial g_{\sigma\mu}}{\partial x^{\nu}}$.

b. Using tensor formalism prove the following vector identity:

$\vec{a}.(\vec{b} \times \vec{c}) = \vec{b}.(\vec{c} \times \vec{a})$

c. Write down the components of metric tensor in spherical polar coordinate. Show that, $dg_{\alpha\beta} = -g_{\mu\alpha}g_{\nu\beta}dg^{\mu\nu}$.

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M.Sc. PHYSICS
FIRST SEMESTER
MATHEMATICAL PHYSICS-I
MSP-101

(Use separate answer scripts for Objective & Descriptive)

Duration: 3 hrs. **Full Marks: 70**

(PART-A: Objective)

Time: 20 min. **Marks: 20**

Choose the correct answer from the following: ***1x20=20***

1. If f(z) is analytic function and f'(z) is continuous at each point within and on the curve 'C' then:
   a. $\int_C zf(z)\,dz = 0$
   b. $\int_C f(z)\,dz = 0$
   c. $\int_C f(z)\,dz = 1$
   d. $\int_C \frac{f(z)}{z}\,dz = 0$

2. The polar form of the complex number z = x + iy is:
   a. $re^{i\theta}$
   b. $re^{-i\theta}$
   c. $re^{i\frac{\theta}{2}}$
   d. $re^{-i\frac{\theta}{2}}$

3. The argument of the complex number $\frac{1-i}{1+i}$ is:
   a. $\frac{\pi}{2}$
   b. π
   c. $\frac{-\pi}{2}$
   d. 2π

4. The value of the integral $\int_C \frac{\sin Z}{z}\,dz$ is:
   a. 1 b. -1 c. 0 d. Π

5. Laplace transformation of the Dirac delta function δ (x-a) (if $e^{isx} = f(x)$) is:
   a. $\frac{1}{\pi}e^{isa}$
   b. $\frac{1}{\sqrt{2\pi}}e^{isa}$
   c. $\frac{1}{2\pi}e^{isa}$
   d. $\frac{1}{\sqrt{2\pi}}e^{-isa}$

6. The general form of the Sturm-Lioville equation (where p, q, r and r' are continuous on [a, b], with p(x)>0 on [a, b] and r(x) > 0 on [a, b]) is:
   a. $[r(x)y]' + [q(x) + \lambda p(x)]y = 0$
   b. $[r(x)y] + [q(x) + \lambda p(x)]y = 0$
   c. $[r(x)y']' + [q(x) + \lambda p(x)]y = 0$
   d. $[r(x)y']' + [q(x) + \lambda p(x)]' y = 0$

7. Residue of $f(z) = \frac{(z+1)}{(z-1)(z-2)}$ at z=1 is:
   a. -2 b. 1 c. 4 d. 2

8. The value of the integral $\int_0^2 (x^3 + 3x + 2)\delta(x - 3)dx$ is:
a. 16 b. 38 c. 0 d. 2

9. If two vectors X and Y are orthogonal, then they satisfy the following relation:
a. $X.Y = 0$
b. $X.Y = 1$
c. $X.Y = \frac{1}{2}$
d. $X.Y = \frac{\pi}{2}$

10. If a matrix A follows the condition $A\bar{A}' = \bar{A}'A = I\ (Identity)$, then the matrix is said to be:
a. Transpose b. Inverse c. Orthogonal d. Unitary

11. The necessary and sufficient condition for matrix A to be Hermitian is
a. $A = \bar{A}$
b. $A = \bar{A}^T$
c. $A = -A^T$
d. $A = A^{-1}$

12. If A and B are two square matrices of same order, and if there exist a non-singular matrix P, then similarity transformation holds the following relation:
a. $B = AP$
b. $B = AP^{-1}$
c. $B = P^{-1}AP$
d. $B = (PA)^{-1}P$

13. The eigen values of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are:
a. 1, 0 b. 0, 2 c. 1, 2 d. 1, 1

14. Sum of the eigen values of a matrix is equal to the sum of the.............. elements.
a. Diagonal
b. Conjugate of diagonal
c. Off diagonal
d. None of these

15. Laplace transform of (e^{2t}) is equal to:
a. $\frac{1}{s}$
b. $\frac{1}{s-2}$
c. $\frac{a}{s^2+a^2}$
d. $\frac{s}{(s-a)^2}$

16. Fourier transform of integral, $F\left[\int_0^t f(t)\,dt\right]$ is equals to:
a. $i\omega F(\omega)$
b. $\frac{1}{i\omega}F(\omega)$
c. $\frac{i}{\omega}F(i\omega)$
d. $\frac{1}{a}F\left(\frac{\omega}{a}\right)$

17. The number of components of a second rank tensor in an N-dimensional space is:
a. 2^N b. $2N$ c. N^2 d. 2N-1

18. The value of the identity $\varepsilon_{iks}\varepsilon_{mps}$ is:
a. 0 b. +1 c. -1 d. 4

19. If $g_{\mu\nu} = 0$ for $\mu \neq \nu$ and μ, ν, σ are unequal indices then, the value of $\Gamma^{\mu}_{\mu\mu}$ is:
a. 0
b. $\frac{1}{2}\frac{g_{\mu\mu}}{\partial x^{\mu}}$
c. $-\frac{1}{2g_{\nu\nu}}\frac{g_{\mu\mu}}{\partial x^{\nu}}$
d. $\frac{1}{2}\frac{\partial(\log g_{\mu\mu})}{\partial x^{\mu}}$

20. If $A_{\mu\nu}$ is an antisymmetric tensor, the value of its component A_{11} is:
a. 1 b. 0 c. -1 d. 2

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(PART-B : Descriptive)

Time : 2 hrs. 40 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ where c^2=T/m=constant subject to condition: Boundary condition u (0, t) = u (l, t) = 0 and initial condition: u (x, 0) = f(x) and at t = 0 where l = length of the string or wire of wave propagation. 10

2. a. State and prove Taylor's theorem of a complex function. 5+3+2=10

b. Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

c. Prove that $\int_C \frac{e^{-z}}{z^2}dz = -2\pi i$

3. a. State and Prove Sturm-Liouville Theorem (Orthogonality of Eigen function). 6+4=10

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ subject to the condition z (x, 0) = x^2 and z (1, y) = cos y.

4. a. What do you mean by similarity transformation? 1+1+8=10

b. State the theorem of diagonalization of a matrix. Find the eigen values, eigen vectors, model matrix and then diagonalize the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

5. a. Define Hermitian matrix. 1+(2+2)+5=10

b. Prove that:

i. $A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$ is Hermitian, and

ii. $A = \begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 2-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$ is skew-Hermitian.

c. Express the matrix $A = \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -3 & -4i & 5 \end{bmatrix}$ as the sum of Hermitian and skew-Hermitian matrix.

6. Define Laplace transformation and solve the following problems: 1+4+5=10

a. Find the Laplace transformation of $f(t)$ defined as

$$f(t) = \begin{cases} t/k, & when\ 0 < t < k \\ 1, & when\ t > k \end{cases}$$

b. Find the Laplace transform of $(1 + sin2t)$.