(PART-B : Descriptive)

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. State and prove the second shifting theorem for Laplace Transform. Hence find $\mathcal{L}[(t-1)^2 U(t-1)]$. 6+4=10

   b. If $\mathcal{L}[t\sin\omega t] = 2\omega s/(s^2+\omega^2)^2$ then find $\mathcal{L}[\omega t\cos\omega t + \sin\omega t]$.

2. a. If $\mathcal{L}[f(t)] = f(s)$ then show that 3+4+3=10

$$\mathcal{L}[f''(t)] = s^2 f(s) - sf(0) - f'(0)$$

   b. Solve:

   $(D^2+4)y - t$ *under the conditions* $y(0) - y'(0) - 0$

   c. Find $\mathcal{L}^{-1}[\frac{s}{s^2+a^2}]$ by applying Convolution Theorem.

3. State Euler's-Lagrange Theorem. Prove that: 2+8=10

$$F_y - \frac{d}{dx}F_{y'} = 0$$

4. State and prove Euler-Poisson Equation. 3+7=10

5. What is Canonical Form? Write down the Canonical Form of one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ 2+8=10

6. Find the characteristics of: 6+4=10

   a. $y^2 r - x^2 t = 0$

   b. $x^2 r + 2xys + y^2 t = 0$

7. Find the extremal of the functional: 5+5=10

   a. $\int_a^b \left(y + \frac{y^3}{3} \right) dx$

   b. $\int_1^3 (3x - y)dx$ that satisfy the boundary conditions

   $y(1) = 1, y(3) = 9/2$

8. If both the ends of a bar of length a are at temperature zero and the initial temperature is to be prescribed function f(x) in the bar, then find the temperature at a subsequent time t. 10

   Or

   The faces x=0 and x=a of an infine slab are maintained at zero temperature. Given that the temperature u(x,t)=f(x) at t=0.Determine the temperature at a subsequent time t.

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M.Sc. MATHEMATICS
FOURTH SEMESTER
ADVANCED PARTIAL DIFFERENTIAL EQUATION
MSM-402

(Use separate answer scripts for Objective & Descriptive)

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1x20=20***

1. The Laplace Transform of the function t^3 is:
   a. $\frac{3!}{s}$ b. $\frac{\Gamma(4)}{s^n}$
   c. $\frac{s!}{\Gamma(3)}$ d. $\frac{3!}{s^4}$

2. In the Laplace Transform of the function $f(t)$, t takes on:
   a. positive finite values b. negative values
   c. values in the interval $0 \le t < \infty$ d. complex values

3. $\mathcal{L}[f'(t)]$ for $f(t) = t$ is:
   a. $1/s$ b. s^2
   c. 1 d. 0

4. Laplace Transform of $\int_0^t f(u)du$ for $\mathcal{L}[f(t)] = f(s)$ is:
   a. $f(s^2)$ b. $s^2 f(s)$
   c. $f(s)/s$ d. $sf(s) - f(0)$

5. One dimensional Wave equation $r - t = 0$ is:
   a. parabolic b. hyperbolic
   c. elliptic d. none of the above

6. λ quadratic equation of $y^2 r - x^2 t = 0$ is:
   a. $y^2\lambda^2 - x^2 = 0$ b. ${}^2\lambda - x^2 = 0$
   c. ${}^2\lambda^2 + x^2 = 0$ d. ${}^2\lambda^2 - x = 0$

7. $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')dx$ is said to be:
   a. integral b. constant
   c. functional d. variable

8. In Euler's Equation ∂y is known as:
   a. parameter b. function
   c. constant d. variation

9. In Euler-Lagrange Equation, the condition for x is:

a. $x_1 \le x \le x_2$
b. $x_1 > x \le x_2$
c. $x_1 \le x > x_2$
d. $x_1 \le x = x_2$

10. In Lagrange equation we consider $F(x, y, y)$ is:

a. differentiable
b. twice differentiable
c. continuous
d. integrable

11. What is value of $\left(\frac{d\phi}{d\alpha}\right)_{\alpha=0}$ is known as:

a. variation
b. variation of the functional
c. function of variation
d. function

12. Which is correct of the following?

a. $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y'}\right) = 0$
b. $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y''}\right) = 0$
c. $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y''}\right) = 0$
d. $\frac{\partial F}{\partial y} + \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y''}\right) = 0$

13. A necessary condition for extremum is:

a. variation decreases
b. variation vanishes
c. variation increases
d. none of the above

14. The Heat Equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\left(\frac{\partial u}{\partial t}\right)$

a. two dimensional
b. three dimensional
c. both two and three
d. one dimensional

15. The solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\left(\frac{\partial u}{\partial t}\right)$ be of the form:

a. $u(x,t) = X(x)T(t)$
b. $u(x,t) = X(t)T(t)$
c. $u(x,t) = X(x)T(x)$
d. $u(x,t) = X(x)$

16. The characteristics equation of $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ are:

a. $\frac{dy}{dz} + \lambda_1 = 0$, $\frac{dy}{dx} + \lambda_2 = 0$
b. $\frac{dy}{dx} + \lambda_1 = 0$, $\frac{dy}{dx} + \lambda_2 = 0$
c. $\frac{dz}{dx} + \lambda_1 = 0$, $\frac{dy}{dx} + \lambda_2 = 0$
d. $\frac{dy}{dx} - \lambda_1 = 0$, $\frac{dy}{dx} - \lambda_2 = 0$

17. In $R\lambda^2 + S\lambda + T = 0$ if $S^2 - 4RT > 0$ then λ value:

a. real
b. real and distinct
c. complex
d. distinct and unequal

18. For reducing a hyperbolic equation to its canonical form we take two variable u and v which are:

a. dependent
b. independent
c. both
d. none of the above

19. In one parameter family of curves in Euler's equation $y(x,\alpha) = y(x) + \alpha[\bar{y}(x) - y(x)]$, the value of α are:

a. 0,3
b. 0,0
c. 1,-1
d. 0,1

20. The partial differential equation of second degree can be classified as:

a. two
b. three
c. four
d. five

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