6. a. Define cutpoint and block of a graph. Prove that if v is a cutpoint of a graph G, then there exists points u and w distinct from v such that v is in every $u - w$ path. 4+3+3=10

b. Define tree. Prove that if G is a tree, then every point of G is joined by a unique path.

c. What is cycle rank of a connected graph? Find the cycle rank of:

(i) K_p (ii) $K_{m,n}$

7. a. Define point connectivity and line connectivity of a graph. 2+6+2=10

b. Prove that for any graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$, where $\kappa, \lambda, \partial$ have their usual meanings.

c. Construct a graph with $\kappa = 3, \lambda = 4, \delta = 5$.

8. a. Define Eulerian graph. Prove that if G is Eulerian, then every point of G has even degree. 3+2+3+2=10

b. Define Hamiltonian graph with an example.

c. Give examples of a graph that is:

(i) Hamiltonian but not Eulerian.

(ii) Eulerian but not Hamiltonian.

(iii) Both Eulerian and Hamiltonian.

d. Let G_1 and G_2 be two Eulerian graphs with no point in common. Let $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$. Let G be the graph obtained from $G_1 \cup G_2$ by adding the lines $v_1 v_2$. What can be said about G?

= = *** = =

M.Sc. MATHEMATICS
FOURTH SEMESTER
GRAPH THEORY
MSM-401

(Use separate answer scripts for Objective & Descriptive)

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

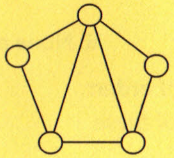
Choose the correct answer from the following: ***1x20=20***

1. The cycle rank of the graph K_{mn} is:
   a. $(m+1)(n+1)$ b. mn
   c. $(m-1)(n+1)$ d. $(m-1)(n-1)$

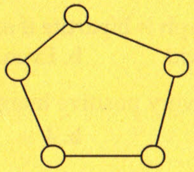
2. Consider the following statements:
   P: Every connected graph has only one spanning tree.
   Q: The collection of all co boundaries of any graph form a vector space over $\{0,1\}$.
   a. Only P is true b. Only Q is true
   c. Both P and Q are true d. Both P and Q are false

3. The chromatic polynomial of the complete bipartite graph $K_{2,s}$ is:
   a. $k(k-1)^s + (k-1)(k-2)^s$ b. $k(k+1)^s + k(k-1)(k-2)^s$
   c. $k(k-1)^s + k(k-1)(k-2)$ d. $k(k-1)^s + k(k-1)(k-2)^s$

4. Which of the following is uniquely 3-colorable graph?
   a.



   b.



   c. Both (a) and (b) d. None of these

5. Consider the following statements:
   P: Every dominating set contains at least one minimal dominating set.
   Q: Every edge covering contains a minimal edge.
   a. P is true, Q is false b. Both P and Q are false
   c. Q is true, P is false d. Both P and Q are true

6. Consider the following statements:
   P: $\chi'(K_n) = \begin{cases} n, & if\ n\ is\ even \\ n-1, & if\ n\ is\ odd \end{cases}$
   Q: $\chi(K_n) = n$
   a. Only P is true b. Only Q is true
   c. Both P and Q are true d. Both P and Q are false

7. Chromatic polynomial of the complete bipartite graph $K_{n,m}$ is:
   a. 2 b. max (n,m)
   c. min (n,m) d. lcm (n,m)

8. If G_1 and G_2 are regular graphs, then consider the following:
(i) $G_1 + G_2$ is regular (ii) $G_1 \times G_2$ is regular
a. (i) is true but (ii) is not true
b. (ii) is true but (i) is not true
c. Both (i) and (ii) are true
d. Neither (i) nor (ii) is true

9. If $u, v \in V(G)$, then eccentricity of v is:
a. $d(u,v)$
b. $Min\ d(u,v)$
c. $Max\ d(u,v)$
d. None of these

10. A maximal complete subgraph of a graph is called:
a. Clique
b. Line graph
c. Spanning graph
d. None of these

11. Which of the following statement is not true?
a. If G is an Eulerian graph, then every point of G has even degree.
b. A cycle C_n, $n \geq 3$ is Eulerian.
c. A complete graph K_n, $n \geq 3$ is Eulerian.
d. None of these.

12. Consider the following statements.
(i)Every subgraph of a planar graph is planar.
(ii) A graph contains a subgraph homeomorphic to $K_{3,3}$ is planar.
a. (i) is true nut (ii) is not true
b. (ii) is true but (i) is not true
c. Both (i) and (ii) are true
d. Neither (i) nor (ii) is true

13. If $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are any two graphs, then the number of lines in $G_1 + G_2$ is:
a. $p_1q_1 + p_2q_2 + 2$
b. $p_1 + p_2 + q_1q_2$
c. $q_1 + q_2 + p_1p_2$
d. None of these

14. A graph is bipartite if and only if all its cycles are:
a. Odd b. Even c. Any positive integer d. None of these

15. For every positive integer $n \geq 4$, there exists a graph of order:
a. 3 b. 4 c. 5 d. None of these

16. Every tree has a centre consisting of either:
a. One or two adjacent points
b. Two or three adjacent points
c. Only one point
d. No centre point

17. From which of the following degree sequence we can construct a graph?
a. 4,4,4,3,2 b. 3,3,3,3,2 c. 2,1,2,1,2,1 d. None of these

18. The point connectivity of a disconnected graph is:
a. 0 b. 1 c. 2 d. 3

19. The maximum vertex of a graph G is Δ, then:
a. $\chi(G) \leq \Delta + 1$
b. $\chi(G) \geq \Delta + 1$
c. $\chi(G) = \Delta + 1$
d. None of these

20. A non separable graph is connected, non trivial and ___________cut point.
a. One
b. Two
c. Zero
d. None of these

-- --- --

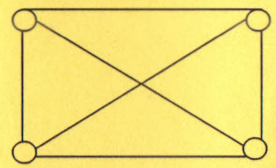
(PART-B : Descriptive)

Time : 2 hrs. 40 min. Marks : 50

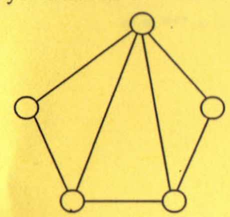
[Answer question no.1 & any four (4) from the rest]

1. a. Define spanning subgraph and induced subgraph of a graph. Give example. 3+3+4=10
b. Define degree of a point of a graph. State and prove the Handshaking theorem.
c. Every (p,q) graph with $q \geq p$ contains a cycle.

2. a. Prove that – the number of n verities of a full binary tree is odd and the number of pendent verities of the tree is equal to $\frac{n+1}{2}$. 5+5=10
b. Define spanning tree. Find all the spanning trees of the following graph:



3. a. Verify with example that – In a graph $G(V, E)$ with n vertices, $S \subseteq V$is a maximum independent set if $\bar{S}$ is a minimum vertex cover, and hence $\alpha_0(G) + \alpha_0'(G)$, where $\alpha_0(G)$ and $\alpha_0'(G)$ denotes the independence and vertex covering numbers respectively. 5+5=10
b. Find the chromatic polynomial of the following graph:



4. a. Determine the number of vertices and edges in a tree having $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. 5+5=10
b. For a graph $G(p,q)$, prove that G is a tree if G is connected, a-cyclic and $p = q + 1$.

5. a. Find the product and composition of the following graphs: 4+3+3=10
(i) $\overline{K_4}$ and K_2
(ii) P_3 and P_2
b. Define planar graph. If $G(p,q)$ is a simple connected planar graph having k edges in a boundary of each region, then show that $(k-2)q \leq k(p-2)$.
c. Show that the graphs K_5 and $K_{3,3}$ are non planar.