(PART-B : Descriptive)

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. What is Laplace Equation.Find Laplace Equation in plane polar coordinate. 2+8=10

2. a. State Cauchy problem for first order partial differential equations. 4+6=10
   b. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ into it's canonical form.

3. a. Prove the Principle of Invariance of Euler's equation under co-ordinate transform. 4+6=10
   b. Apply the method of separation of variables to find a solution of the form $u = A(p)e^{ip(t\pm\frac{x}{c})} for\ the$ one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x}$, for arbitrary forms of the function A.

4. a. State Uniqueness Theorem. 4+6=10
   b. Prove that $y_n = y_0 + \sum_{n=1}^{n}(y_n - y_{n-1})$ must be continuous

5. a. Prove that $F(\alpha;\beta;\gamma;x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt$ 5+5=10
   b. State and proof Gauss Theorem

6. a. Find the eigen values and the corresponding eigenfunctions of 10
   $X'' + \lambda X = 0, X(0) = 0, X'(L) = 0$

7. What is Singular Solution.Find the Singular Solution of the equation $1 + (y')^2 = \frac{1}{y^2}$ 2+8=10

8. a. Find the general solution of the equation 5+5=10
   $$(D^2 + 2DD' + D'^2)z = e^{2x+3y}$$
   b. From among the curves connecting the points $P_1(1,3)$ and $P_2(2,5)$, find the one on which an extremum of the functional $\int_1^2 y'(1 + x^2 y')\,dx$ can be attained.

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M. Sc. MATHEMATICS
SECOND SEMESTER
DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS
MSM – 202

(Use Separate Answer Scripts for Objective & Descriptive)

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1×20=20***

1. Which of the following equations is hyperbolic?
   a. $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$ b. $x^2 r - 2xys + y^2 t - xp + 3yq = 0$
   c. $r - 2yp + y^2 z = (y-2)e^{2x+3y}$ d. $r + 2s + t = 0$

2 Find out the nonlinear partial differential equation
   a. $p + q = z + xy$ b. $yp + xq = \frac{x^2z^2}{y^2}$
   c. $x^2p^2 + y^2q^2 = z^2$ d. $x^2zp + y^2zq = xy$

3. The functional $I[y(x)] = \int_{x_1}^{x_2}[p(x)y + q(x)y']dx$ is
   a. Linear b. Non-linear
   c. Composite d. Multiple

4. The functional $\int_0^1 \sqrt{1 + \{y'(x)\}^2}\,dx$ for $y(x) = 1$ has the value
   a. 0 b. 1
   c. 2 d. 10

5. A geodesic is a
   a. circle of minimum circumference b. plane section of solid cone
   c. curve on a surface with minimal condition between two fixed points. d. hyperbolic curve

6. In the equation $F(D, D')z = 0$, D is
   a. A variable b. An operator
   c. A function of x d. A function of y

7. The most general solution for z of the equation $F(D, D')z = f(x,y)$ is
   a. a complex function b. the sum of two particular integrals
   c. the sum of complementary function and particular integral d. an infinite series

8. In the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=a^2 \frac{\partial^2 u}{\partial x}$ the variable u varies in the

a. xy – plane
b. yz – plane
c. xyz – space
d. $x - t$ space

9. The Pochhammer symbol is denoted by

a. $(\alpha)_n = \alpha(\alpha+1).....(\alpha+n-1)$
b. $(\alpha)_n = \alpha(\alpha-1).....(\alpha+n-1)$
c. $(\alpha)_n = \alpha(\alpha+1).....(\alpha+n)$
d. $(\alpha)_n = \alpha(\alpha+1).....(\alpha+n+1)$

10. In Existence and Uniqueness theorem the function f(x,y) is

a. Differentiable
b. continuous
c. Twice differentiable
d. exist

11. P-discriminant is in involved in the system of equation

a. $F(x,y,y')=0$, $\frac{\partial F(x,y,y'')}{\partial y'}=0$
b. $F(x,y,y'')=0$, $\frac{\partial F(x,y,y')}{\partial y'}=0$
c. $F(x,y,y')=0$, $\frac{\partial F(x,y,y')}{\partial y}=0$
d. $F(x,y,y')=0$, $\frac{\partial F(x,y,y')}{\partial y'}=0$

12. Node locus included in

a. P-disc
b. c-disc
c. Both
d. None of the above

13. Which is correct of the following

a. $F(\alpha;\beta;\gamma;x)=F(\alpha;\beta;1;x)$
b. $F(\alpha;\beta;\gamma;x)=F(\beta;\alpha;1;x)$
c. $F(\alpha;\beta;\gamma;x)=F(\beta;\alpha;\gamma;x)$
d. none of the above

14. What is value of $(\alpha)_0 = ?$

a. 1
b. 2
c. 0
d. 3

15. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ is a Laplace Equation

a. One Dimension
b. Two dimension
c. Three Dimension
d. none

Contd....

16. What is value of $\frac{d}{dx}F(\alpha;\beta;\gamma;x)=?$

a. $\frac{\alpha\beta}{\gamma}F(\alpha+1;\beta+1;\gamma+1;x)$
b. $\frac{\alpha\beta}{\gamma}F(\alpha;\beta;\gamma;x)$
c. $\frac{\alpha\beta}{\gamma}F(\alpha+1;\beta+1;\gamma+1;1)$
d. $\frac{\alpha\beta}{\gamma}F(\alpha+1;\beta;\gamma+1;x)$

17. $\frac{\partial^2 v}{\partial r^2}+\left(\frac{1}{r}\right)\left(\frac{\partial v}{\partial r}\right)+\left(\frac{1}{r^2}\right)\left(\frac{\partial^2 v}{\partial\theta^2}\right)+\left(\frac{\partial^2 v}{\partial z^2}\right)=0$ is

a. Wave equation
b. Heat equation
c. Laplace equation
d. Heat equation in polar form

18. $(\alpha+n)(\alpha)_n = ?$

a. $(\alpha+n)(\alpha)_{n+1}$
b. $(\alpha)_{n+1}$
c. $(\alpha+n)$
d. $(\alpha+n)(\alpha)$

19. Existence theorem is called existence because it does have

a. No value
b. One value
c. value
d. none

20. C-disc factorised as

a. $E\times T\times C$
b. $E\times N^2\times C$
c. $E\times T^2\times C$
d. $E\times N^2\times C^3$

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P.T.O.