(PART-B : Descriptive)

Time: 2 hrs. 40min. Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. Write the statement of Uniqueness and Existence Theorem. Solve by Picard's Theorem. 3+7=10

$$\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right) \text{ where } y = 1, \frac{dy}{dx} = \frac{1}{2} \text{ when } x = 0$$

2. Solve: 5+5=10

   a. $z = ax + by + ab$

   b. $\lambda^2 = (x-h)^2 + (y-k)^2 + z^2$

3. What is total Differential equation? 1+9=10
   Solve:

$$(x+1)\frac{d^2y}{dx^2} - 2(x+3)\frac{dy}{dx} + (x+5)y = e^x$$

4. State and Proof Legendre Polynomial of 1st kind. 2+8=10

5. Define Lipschitz condition. Prove that $y_n = y_0 + \sum_{n=1}^{n}(y_n - y_{n-1})$ must be continuous. 2+8=10

6. Write two differences between Simultaneous and Total differential equation. Solve by Auxillary method. 6+4=10

$$(y^2 + yz + z^2)dx + (z^2 + zx + x^2)dy + (x^2 + xy + y^2) = 0$$

7. a. State and Proof Gauss Hypergeometric function. 4+6=10

   b. Prove that $\frac{d}{dx}F(\alpha;\beta;\gamma;x) = \frac{\alpha\beta}{\gamma}F(\alpha+1;\beta+1;\gamma+1;x)$

8. What is Ordinary and Singular Points. Discuss the singularity of the equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ at x=0 and $x = \infty$. 3+7=10

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M.Sc. MATHEMATICS
FIRST SEMESTER
DIFFERENTIAL EQUATION
MSM-102

(Use separate answer scripts for Objective & Descriptive)

Duration : 3 hrs. Full Marks : 70

(PART-A : Objective)

Time : 20 min. Marks : 20

Choose the correct answer from the following: *1×20=20*

1. $Pdx + Qdy + Rdz = 0$ is a total differential equation, where R is a function of:

   a. y b. z
   c. x d. x, y

2. $y = x$ is a part of complementary function of the differential equation of 2nd order if:

   a. $P + Qx = 0$ b. $1 + P + Q = 0$
   c. $P + Qx + 2 = 0$ d. $P + Qx^2 = 0$

3. In the statement of Existence and Uniqueness theorem $f(x, y)$ is a:

   a. Real function b. Imaginary function
   c. Continuous function d. Analytic function

4. In Picard's theorem nth approximation is given by:

   a. $y_n(x) = \int_{x_0}^{x} f(x, y)dx$ b. $y_n(x) = \int_{x_0}^{x} f(x, y_{n-1})dx$
   c. $y_n(x) = y_0 - \int_{x_0}^{x} f(x, y_{n-1})dx$ d. $y_n(x) = y_0 + \int_{x_0}^{x} f(x, y_{n-1})dx$

5. In the method of Removal of first derivative:

   a. $Y = \mathrm{Re}^{\frac{1}{2}\int Pdx}$ b. $Y = \mathrm{Re}^{-\frac{1}{2}\int Pdx}$
   c. $Y = Qe^{-\frac{1}{2}\int Pdx}$ d. $Y = e^{-\frac{1}{2}\int Pdx}$

6. One of solution of $\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$ is:

   a. $x^3 + y^3 = c_1$ b. $x^3 + y^3 = c_1$
   c. $x^2 + y^2 = c_1$ d. $x^2 - y^2 = c_1$

7. Lipschitz Condition is:

   a. $|f(x, y_1) - f(x, y_2)| \ge k|y_1 - y_2|$ b. $|f(x, y_1) - f(x, y_2)| = k|y_1 - y_2|$
   c. $|f(x, y_1) - f(x, y_2)| \le k|y_1 - y_2|$ d. $|f(x, y_1) - f(x, y_2)| < k|y_1 - y_2|$

8. Value of Legendre Polynomial $P_0(1) = ?$

a. 1 b. -1 c. 0 d. 2

9. In the method of variation of Parameter, $y = Au + Bv$ is the C.F of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$,where:

a. u, v are arbitrary constant
b. u, v are function of x
c. v variable
d. u variable

10. Existence Theorem is called existence, because the Initial Value Problem:

a. Has a constant value
b. Has no solution
c. Is variable
d. Does have a solution

11. Generating function for Legendre Polynomial is defined by:

a. $(1-2xz+z^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n P_n(x)$
b. $(1-2xz+z^2)^{-1} = \sum_{n=0}^{\infty} z^n P_n(x)$
c. $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} zP_n(x)$
d. $(1-2xz+z^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n P(x)$

12. One of the condition of Integrability of $Pdx + Qdy + Rdz + Tdt = 0$ is:

a. $P\left(\frac{\partial Q}{\partial t} - \frac{\partial T}{\partial y}\right) - Q\left(\frac{\partial T}{\partial x} - \frac{\partial P}{\partial t}\right) + T\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
b. $P\left(\frac{\partial Q}{\partial t} - \frac{\partial T}{\partial y}\right) + Q\left(\frac{\partial T}{\partial x} - \frac{\partial P}{\partial t}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
c. $P\left(\frac{\partial Q}{\partial t} - \frac{\partial T}{\partial y}\right) + Q\left(\frac{\partial T}{\partial x} - \frac{\partial P}{\partial t}\right) + T\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
d. $P\left(\frac{\partial Q}{\partial t} - \frac{\partial T}{\partial y}\right) + Q\left(\frac{\partial P}{\partial x} - \frac{\partial T}{\partial t}\right) + T\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

13. A Linear Differential Equation of 2nd Order is of the form:

a. $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$
b. $\frac{d^2y}{dx^2} - P\frac{dy}{dx} + Qy = R$
c. $\frac{d^2y}{dx^2} - P\frac{dy}{dx} + Qy = 0$
d. $\frac{d^2y}{dx^2} - P\frac{dy}{dx} - Qy = R$

14. Order of $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy$ is:

a. 2 b. 3 c. 0 d. 1

15. $x^2y'' + xy' + (x^2 - n^2)y = 0$ where n being non-negative constant:

a. Legendre Equatin
b. Bessel Equation
c. Hypergeomeric Equation
d. Power series

16. In Pochhammer symbol $(\alpha)_0 = \alpha(\alpha+1)\ldots\ldots\ldots(\alpha+n-1)$

where:

a. n parameter
b. n positive constant
c. n Negative constant
d. n variable

17. In Partial derivatives $\frac{\partial^2 z}{\partial x^2} = ?$

a. r b. p c. q d. s

18. Confluent hypergeometric function is defined by:

a. $F(\alpha; x) = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{(\beta)_r}\frac{x^r}{r}$
b. $F(\alpha;\beta;x) = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{(\beta)_r}\frac{x}{r}$
c. $F(\alpha;\beta;x) = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{(\beta)_r}\frac{x^r}{r}$
d. $F(\alpha;\beta;x) = \sum_{r=1}^{\infty} \frac{(\alpha)_r}{(\beta)_r}\frac{x^r}{r}$

19. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$

a. Is a non-linear PDE
b. LINEAR ODE
c. Non-linear ODE
d. Linear PDE

20. $(\alpha + n)(\alpha)_n = ?$

a. $(\alpha)_{n+1}$
b. $(\alpha)_n$
c. $(\alpha)_{n-1}$
d. $(\alpha)_0$

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