REV-00
MSM/17/22

2016/12

M.Sc. MATHEMATICS
First Semester
ADVANCED ABSTRACT ALGEBRA
(MSM - 101)

Duration: 3Hrs. **Full Marks: 70**

Part-A (Objective) =20
Part-B (Descriptive) =50

(PART-B: Descriptive)

Duration: 2 hrs. 40 mins. **Marks: 50**

Answer any *four* from *Question no.* 2 to 8
***Question no.* 1 is compulsory.**

1. (a) State and prove Jordan-Holder theorem.

   (b) Prove that for two ideals A and B of a ring R, $A \cup B$ is an ideal of R if and only if either $A \subseteq B$ or $A \supseteq B$.

(6+4=10)

2. (a) Define normal and subnormal series for a group G. Give an example of a subnormal series which is not normal series of G.

   (b) Let G be a group and N be normal subgroup of G. Prove that if N and G/N are solvable groups, then G is also a solvable group.

(5+5=10)

3. (a) State and prove the fundamental theorem of group homomorphism.

   (b) Let $G = \{1, -1\}$ be a group under multiplication. Define $f : \mathbb{Z} \to G$ by putting

$$f(n) = \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases}.$$

   Prove that f is a homomorphism of $\mathbb{Z}$ into G.

(6+4=10)

4. (a) State Sylow's second theorem. Prove that if a group G contains a single subgroup of a certain order then that subgroup is normal in G.

(b) State Eisenstein's irreducibility criterion for a polynomial f. Using that criterion check whether following polynomials are irreducible or not in $\mathbb{Q}[x]$.

x^4-4x+2

$x^3-9x+15$

$7x^4-2x^3+6x^2-10x+18$

(5+5=10)

5. (a) Prove that an ideal M of a commutative ring R with unity is a maximal ideal if and only if R/M is a field.

(b) What is a prime ideal? Give one example.

(8+2=10)

6. (a) What is a prime element in a ring? Prove that in $\mathbb{Z}[\sqrt{-5}]=\{a+b\sqrt{-5}:a,b\in\mathbb{Z}\}$, $\sqrt{-5}$ is a prime element.

(b) Define PID. Prove that $\mathbb{Z}$ is a PID.

(5+5=10)

7. (a) What is a simple extension of a field? Give one example.

(b) What is an algebraic extension of a field? Show that $\sqrt{1+\sqrt{3}}$ is algebraic over $\mathbb{Q}$.

(c) Show that x^3-2 is the minimal polynomial of $\sqrt[3]{2}$ over $\mathbb{Q}$.

(2+5+3=10)

8. (a) Let R be a commutative ring, F be the ring of right endomorphism of the additive Abelian group A. If $\Gamma:R\to F$ is a ring homomorphism, then prove that by putting $ar=a(\Gamma r)$ for all $a\in A$ and $r\in R$, we obtain a right R-module A_R, and every right R-module can be obtained in this way. Conversely, if we have a right R-module, then $\Gamma:R\to F$ is a ring homomorphism.

(b) Define a module over a ring.

(8+2=10)

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Duration: 20 minutes **Marks – 20**

(PART A - Objective Type)

I. Choose the correct answer: **1×20=20**

1. For a finite group G and for each normal subgroup N of G, $o\left(G/N\right)=?$
   (a) $o(G)+o(N)$ (b) $o(G)/o(N)$
   (c) $o(G)o(N)$ (d) $o(G)-o(N)$

2. A homomorphism f, which is at the same time is also onto is called
   (a) Monomorphism (b) Endomorphism
   (c) Epimorphism (d) Automorphism

3. Let N be a normal subgroup of a group G, then the mapping $f: G \to G/N$ is defined by $f(x)=Nx \quad \forall x \in G$ is called the
   (a) Natural homomorphism (b) Canonical homomorphism
   (c) Both (a) and (b) (d) None

4. Let G be a group of order 200. Then G is
   (a) An abelian group (b) A simple group
   (c) Not a simple group (d) All of the above

5. If G is a finite abelian group and a positive integer $k$divides$o(G)$, then G contains a subgroup of order
   (a) 1 (b) k (c) $k/2$ (d) k^2

6. Let G be a group and $H \leq G$. If $p^k / o(G)$ and $p^k \nmid o(G)$, then any subgroup H of order p^k is called
   (a) p-subgroup (b) Sylow subgroup
   (c) Sylow$p$-subgroup (d) None

7. Every group of order p^2, where p is a prime, is ***abelian/non-abelian***. (Pick the correct one)

8. Which of the following statement is correct?
   (a) $\mathbb{Z}$ is a subring of $\mathbb{Q}$. (b) $\mathbb{Z}$ is an ideal of $\mathbb{Q}$.
   (c) $\mathbb{Z}$is a left ideal of $\mathbb{Q}$. (d) $\mathbb{Z}$ is a right ideal of $\mathbb{Q}$.

9. A division ring is a
   (a) Commutative ring (b) Non-commutative ring
   (c) Simple ring (d) none

10.For any two ideals A and B of a ring $A+B$=?

(a) $\langle A \cap B \rangle$ (b)$\langle A - B \rangle$ (c)$\langle A \backslash B \rangle$ (d) $\langle A \cup B \rangle$

11.Let R be a ring and A be an ideal of R. If $A^n = (0)$ for some positive integer n, then A is

(a) Nil ideal (b) Nilpotent ideal
(c) Right ideal (d) Left ideal

12.Consider the ideal (0) of $\mathbb{Z}$. Then (0) is a

(a) Maximal ideal of $\mathbb{Z}$ (b) Not a prime ideal of $\mathbb{Z}$
(c) Prime ideal of $\mathbb{Z}$ (d) None

13.The ideal (4) in E, the ring of even integers is a

(a) Maximal ideal (b) Not a maximal ideal
(c) Prime ideal (d) None

14.Le A and B be two ideals of a ring R, then $\frac{A+B}{A} \cong ?$

(a) $\frac{A+B}{B}$ (b) $\frac{A \cap B}{B}$ (c)$\frac{A \cap B}{A}$ (d) $\frac{B}{A \cup B}$

15.In$\mathbb{Z}/_{(8)}$, the associates of $\bar{2}$ are

(a) $\bar{3}$ and $\bar{4}$ (b) $\bar{2}$ and $\bar{6}$ (c) $\bar{2}$ and $\bar{4}$ (d) $\bar{2}$ and $\bar{0}$

16.In$\mathbb{Z}/(6)$, $\bar{2}$ is

(a) An irreducible element (b) A unit element
(c) A prime element (d) Unity element

17.In the ring $\mathbb{Z}/_{(12)}$, HCF of $\bar{6}$ and $\bar{8}$ is

(a) $\bar{1}$ (b) $\bar{0}$ (c)$\bar{3}$ (d) $\bar{2}$

18.Which of the following statement is correct?

(a) Every Euclidean domain is principal ideal domain.
(b) Every principal ideal domain is Euclidean domain.
(c) Every unique factorization domain is principal ideal domain.
(d) Every unique factorization domain is Euclidean domain.

19.The polynomial $x^3 - 9x + 15$ is .... polynomial in $\mathbb{Q}[x]$.

(a) A reducible (b) A minimal
(c) An irreducible (d) None

20. $[\mathbb{Q}[\sqrt{2}]:\mathbb{Q}] = ?$

(a) 0 (b) 1 (c) -1 (d) 2

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