M.Sc. MATHEMATICS
THIRD SEMESTER
FUNCTIONAL ANALYSIS
MSM-302

Duration: 3 Hrs. **Marks: 70**

PART : A (OBJECTIVE) = 20
PART : B (DESCRIPTIVE) = 50

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. (i) Define Banach space. (2+8=10)
   (ii) Show that $C[a,b]$ is a Banach space under the norm
   $$\| f \| = \sup\{| \ f(x)| : a \le x \le b\},$$
   Where $C[a,b]$ is the set of all continuous functions from $[a,b] \to R$

2. Define convex set. If M is a proper closed subspace of a Hilbert space H, then there exist a non-zero vector z_0 in H such that $z_0 \perp M$. (2+8=10)

3. (i) Give the statement of Hahn Banach Theorem. (4+6=10)
   (ii) If N is a normed linear space and x_0 is a non zero vector in N, then there exists a functional f_0 in N^* such that $f_0(x_0) = \| x_0 \|$ and $\| f_0 \| = 1$.

4. Let P be a projection on a Banach space X. Then show that (5+5=10)
   (i) $I - P$ is a projection on X
   (ii) $R(P) = \{x \in X : Px = x\}$
   (iii) $R(P) = N(I - P)$
   (iv) $X = R(P) \oplus N(I - P)$
   (v) If P is bounded then $R(P)$ and $R(I - P)$ are closed.

5. (i) Define isometric isomorphism and homeomorphism. (4+6=10)
   (ii) Prove that the translation $T_a : X \to X$ such that $T_a(x) = a + x$ is a homeomorphism.

6. Prove that: (5+5=10)

   (i) A compact subset M of a normed linear space is closed and bounded.

   (ii) A subspace M of a Banach space X is a Banach space iff M is closed in X.

7. Define orthogonal set. If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then prove that the linear subspace M+N is also closed. (2+8=10)

8. Define perpendicular projection. If P is a bounded projection on H with R(P)=M and N(P)=N, then $M \perp N \Leftrightarrow$ P is self adjoint and N= $M^{\perp}$ (2+8=10)

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[PART-A : Objective]

Choose the correct answer from the following: 1×20=20

1. Let N be a normed linear space and $x, y \in N$, then which of the following is true?
   a. $|\| x \| - \| y \|| \geq \| x - y \|$
   b. $|\| x \| - \| y \|| \leq \| x - y \|$
   c. $|\| x \| - \| y \|| = \| x - y \|$
   d. None of these

2. In a normed linear space:
   a. $\| -x \| = \| x \|$
   b. $\| -x \| = - \| x \|$
   c. $\| -x \| = | x |$
   d. None of these

3. A linear transformation $T : (X, \| . \| \rightarrow (Y, \| . \|))$ is continuous if T is:
   a. Bounded
   b. Unbounded
   c. Open map
   d. None of these

4. A normed linear space X is said to be compact if every sequence in X has a:
   a. Convergent subsequence.
   b. Divergent subsequence.
   c. May or may not have a convergent subsequence.
   d. None of these.

5. The identity operator on a normed linear space is:
   a. Bounded.
   b. Unbounded.
   c. May or may not be bounded.
   d. None of these.

6. If T is continuous at the origin, then there exists a real number K such that for all $x \in N$
   a. $\| T(x) \| = K \| x \|$
   b. $\| T(x) \| \geq K \| x \|$
   c. $\| T(x) \| \leq K \| x \|$
   d. None of these

7. A compact subset M of a normed linear space is:
   a. Open and bounded.
   b. Closed and bounded.
   c. Closed and unbounded.
   d. None of these.

8. Every finite dimensional normed linear space is:
   a. Reflexive
   b. Not reflexive
   c. Compact
   d. None of these

9. A map $F : (X, \| . \|) \rightarrow (Y, \| . \|)$ is called an isometry if:
   a. $\| x - y \| = \| F(y) - F(x) \|$
   b. $\| x - y \| = \| F(x) - F(y) \|$
   c. $\| y - x \| = \| F(x) - F(y) \|$
   d. None of these

10. Let B and B' be the Banach spaces and T is a continuous linear mapping from B onto B', then T is a/an:
    a. Open map.
    b. Closed map.
    c. Closed and Bounded map.
    d. None of these.

11. The product of two bounded self-adjoin linear operators S and T on a Hilbert space H is self- adjoint if and only if the operators:
    a. Commute
    b. Equal
    c. Similar
    d. None of these

12. A linear operator P on Banach space is said to be a projection if and only if:
    a. $P = P^*$
    b. $P = P^2$
    c. $PP^* = P^*P$
    d. None of these

13. A bounded linear operator $T : H \rightarrow H$, on a Hilbert space H is unitary if T is ...... and T^* =....
    a. bijective, $T^* = T^2$
    b. bijective, $T^* = T^{-1}$
    c. bijective, $TT^* = T^*T$
    d. None of these

14. Let $T : H \rightarrow H$ be a bounded linear operator on a Hilbert space H. If H is complete and $< Tx, x >$ is real for all $x \in H$, the operator T is:

a. Bijective b. Unitary
c. Normal d. Sefl-adjoint

15. Let $T : H_1 \rightarrow H_2$ be a bounded linear operator, where H_1 and H_2 are Hilbert spaces. Then the Hilbert adjoint operator T^* of T is the operator $T^* : H_2 \rightarrow H_1$ such that $\forall x \in H_1$ and $y \in H_2$, $< Tx, y >=$ ....

a. $< x, T^{-1} y >$
b. $< x, Ty >$
c. $< x, T^* y >$
d. None of these

16. A non-empty subset A of a Hilbert space H is said to be orthonormal if and only if $\forall x, y \in A$

a. $\langle x, y \rangle = 1, \langle x, x \rangle = 0$
b. $\langle x, y \rangle = 0, \langle x, x \rangle = 1$
c. $\langle x, y \rangle = 0, \langle x, x \rangle = 0$
d. None of these

17. If M is closed subspace of a Hilbert space H, then:

a. $H = M \oplus M^{\perp}$
b. $H = M^{\perp}$
c. $H = M + M^{\perp}$
d. None of these

18. A closed convex subset C of a Hilbert space H contains a unique vector of:

a. largest norm b. zero norm
c. smallest norm d. None of these

19. The space l^p with $p \neq 2$, is not an inner product space and hence is:

a. a Hilbert space b. a Banach space
c. not a Hilbert space d. One of these

20. In a Hilbert space H, if s, s_1, s_2 are subsets of H, then:

a. $s^{\perp}$ is a closed subspace of H
b. $s^{\perp}$ is a Banach space
c. $s \subseteq s^{\perp}$
d. None of these

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

[PART (A) : OBJECTIVE]

Duration : 20 Minutes

Serial no. of the main Answer sheet

Course : ....

Semester : .... Roll No : ....

Enrollment No : .... Course code : ....

Course Title : ....

Session : 2017-18 Date : ....

Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- Students shall tick (✓) the correct answer.
- No marks shall be given for overwrite / erasing.
- Students have to submit the Objective Part (Part-A) to the invigilator just after completion of the allotted time from the starting of examination.

Full Marks	Marks Obtained
20	

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*