M.Sc. MATHEMATICS
SECOND SEMESTER
TENSOR & MECHANICS
MSM-204

Duration: 3 Hrs. **Marks: 70**

{ PART : A (OBJECTIVE) = 20
PART : B (DESCRIPTIVE) = 50 }

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. Deduce the expressions of velocity and acceleration in terms of spherical polar coordinate. (3+7=10)

2. Deduce Euler's equation of motion of a body about a fixed point. (10)

3. State Christoffel's bracket of 1st kind and second kind. Prove that $g_{\alpha\beta\ ,\gamma} = 0$ (2+8=10)

4. Write briefly about integrals of Energy and Angular Momentum. (10)

5. Define Torque and Angular Momentum. Prove that rate of change of Angular Momentum is equal to Torque. (10)

6. What is symmetric and anti-symmetric tensor? Show that every contraction reduces the rank of a tensor by two. (2+8=10)

7. A heavy particle of mass m moves on the smooth inner surface of a sphere of radious a and its greatest and least depth below the centre are $\frac{a}{2}$ and $\frac{a}{4}$ such that when the depth below the centre is z ,the normal reaction is $3mg\frac{z+a/2}{a}$ (10)

8. What is Bianchi identity? Prove that $B^{\varepsilon}{}_{\mu\nu\sigma\ ,p} + B^{\varepsilon}{}_{\mu\sigma p\ ,\nu} + B^{\varepsilon}{}_{\mu p\nu\ ,\sigma} = 0$ (3+7=10)

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[PART-A : Objective]

Choose the correct answer from the following: **1×20=20**

1. In $a_i x^i$, i is called
   - a. Real suffix
   - b. Dummy suffix
   - c. Kronecker delta
   - d. Lower suffix

2. For the Lagrangian $L = \frac{1}{2}\dot{q}^2 - q\dot{q} + q^2$, find p conjugate to $\dot{q}$
   - a. $\dot{q} + q$
   - b. $q\dot{q}$
   - c. $q - \dot{q}$
   - d. $\dot{q} - q$

3. For the system with Hamiltonian H=$q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$ which of the following statements is not true?
   - a. $q_1 = Ae^t$
   - b. $q_2 = Be^t$
   - c. $p_1 = -p_1 + 2aq$
   - d. $p_2 = p_1 + 2aq$

4. $\left(\frac{\partial x^i}{\partial x^j}\right)$ known as
   - a. Summation convention
   - b. Kronecker delta
   - c. Real suffix
   - d. Dummy suffix

5. Hamiltonian in general is function of p,q and t i.e
   - a. H=$\sum_i \dot{q}_i p_i - L(q, \dot{q}, t)$
   - b. H=$\sum_i \dot{q}_i p_i + L(q, \dot{q}, t)$
   - c. $H = L(q, \dot{q}, t)$
   - d. H=$\sum_i \dot{q}_i p_i$

6. Hamiltonian system of equation is given by
   - a. $\frac{\partial H}{\partial q_i} = -\frac{\partial L}{\partial q_i}$
   - b. $\frac{\partial H}{\partial q_i} = \frac{\partial L}{\partial q_i}$
   - c. Both of these
   - d. None of these

7. We define _____ to be the generalized momentum associated with generalized coordinates $\dot{q}_\alpha$
   - a. $p_\alpha = -\frac{\partial T}{\partial \dot{q}_\alpha}$
   - b. $p_\alpha = \frac{\partial T}{\partial \dot{q}_\alpha}$
   - c. Both of these
   - d. None of these

8. The angular momentum of a rigid body with one end fixed is given by
   - a. L=$\sum_i m_i\{r_i \times (\vec{w} \times r_i)\}$
   - b. L=$\sum_i m_i\{r_i \times v_i\}$
   - c. Both of these
   - d. None of these

9. Which of the following is true?
   - a. $A_{\mu\nu,\sigma} = \frac{\partial A_{\mu\nu}}{\partial x_\sigma} - \Gamma^\alpha{}_{\mu\sigma} A_{\alpha\upsilon} - \Gamma^\alpha{}_{\upsilon\sigma} A_{\mu\alpha}$
   - b. $A_{\mu\nu,\sigma} = \frac{\partial A_{\mu\nu}}{\partial x_\sigma} - \Gamma^\alpha{}_{\mu\sigma} A_{\alpha\upsilon} - \Gamma^\alpha{}_{\upsilon\mu} A_{\mu\sigma}$
   - c. $A_{\mu\nu,\sigma} = \frac{\partial A_{\mu\sigma}}{\partial x_\upsilon} - \Gamma^\alpha{}_{\mu\sigma} A_{\alpha\upsilon} - \Gamma^\alpha{}_{\upsilon\mu} A_{\mu\sigma}$
   - d. None of these

10. The principle of least action states that the action in the actual path is _____ compared with the varied path.
    - a. Minimum
    - b. Maximum
    - c. Zero
    - d. Null

11. The Moment of Inertia of any cube of side a and mass M has the moment of inertia about any side equal to
    - a. Ma
    - b. $\frac{1}{3}Ma^2$
    - c. $\frac{2}{3}Ma^2$
    - d. $\frac{3}{2}Ma^2$

12. $A'^i = \frac{\partial x'^i}{\partial x^\alpha} A^\alpha$ is a tensor of rank
    - a. 3
    - b. 2
    - c. 0
    - d. 1

13. A rectangular parallelepiped with edges a,b and c along x,y and z axis respectively has the Moment of Inertia about any side equal to
    - a. $\frac{2}{3}M(a^2 + b^2)$
    - b. $\frac{2}{3}M(a^2\, b^2)$
    - c. $\frac{2}{3}M(b^2 + c^2)$
    - d. $\frac{1}{3}M(a^2 + b^2)$

14. According to Carnot's Theorem, The Kinetic energy of a system after explosion is increased by the kinetic energy of a relative motion _____ impulse.
    a. Before b. After
    c. Both of these d. None of these

15. The Lagrange's Equation for a holonomic sysrem is given by $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\alpha}\right) - \left(\frac{\partial T}{\partial q_\alpha}\right)$ equal to
    a. $-\sum_{\vartheta=1}^{N} \overrightarrow{F_\vartheta}.\, dr_\vartheta$
    b. $\sum_{\vartheta=1}^{N} \overrightarrow{F_\vartheta}.\, dr_\vartheta$
    c. $\sum_{\vartheta=1}^{N} \overrightarrow{F_\vartheta} \times dr_\vartheta$
    d. None of these

16. Kronecker delta has value
    a. 1 and 2 b. -1 and -2
    c. 0 and 1 d. Both zero

17. $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) = \left(\frac{\partial L}{\partial q_\alpha}\right)$ is the equation for _____
    a. Lagrange's nonconservative holonomic system
    b. Lagrange's conservative holonomic system
    c. Lagrange's conservative nonholonomic system
    d. None of these

18. The radial acceleration in case of motion of a particle in three dimension with cylindrical polar coordinate is given by
    a. $\ddot{r} + r\dot{\theta}^2$
    b. $\dot{r}^2 - r\dot{\theta}^2$
    c. $\ddot{r} - r\dot{\theta}^2$
    d. $\ddot{r} - r^2\dot{\theta}^2$

19. If product of inertia are zero and the principal moments of inertia is nonzero, then by Kinetic energy of rotation we get the value of T as
    a. $2[Aw_1{}^2 + Bw_2{}^2 + Cw_3{}^2]$
    b. $\frac{1}{3}[Aw_1{}^2 + Bw_2{}^2 + Cw_3{}^2]$
    c. $\frac{1}{4}[Aw_1{}^2 + Bw_2{}^2 + Cw_3{}^2]$
    d. $\frac{1}{2}[Aw_1{}^2 + Bw_2{}^2 + Cw_3{}^2]$

20. The Lagrangian of a system can be written as
    a. Kinetic Energy + Potential Energy
    b. Potential Energy- Kinetic Energy
    c. Kinetic Energy – Potential Energy
    d. None of these

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

Question Paper CUM Answer Sheet

[PART (A) : OBJECTIVE]

Serial no. of the main Answer sheet

Course : .....

Semester : ..... Roll No : .....

Enrollment No : ..... Course code : .....

Course Title : .....

Session : 2016-17 Date : .....

Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- The student shall write the answer in the box where it is provided.
- The student shall not overwrite / erase any answer and no mark shall be given for such act.
- Hand over the question paper cum answer sheet (Objective) within the allotted time (20 minutes / 10 minutes) to the invigilator.

Full Marks	Marks Obtained	Remarks
20		

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*