M.Sc. MATHEMATICS
SECOND SEMESTER
LINEAR ALGEBRA
MSM-203

Duration: 3 Hrs. **Marks: 70**

Part : A (Objective) = 20
Part : B (Descriptive) = 50

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. Define subspace of a vector space. The union of two subspaces of a vector space is a subspace if and only if one is contained in the other. (3+7=10)

2. (a) State and prove the Cayley Hamilton theorem. (2+6+2=10)
   (b) Verify this theorem for the following matrix.
   $$\begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}$$

3. (a) Define adjoint operator. (2+2+2+2+2=10)
   (b) Prove the following:
   (i) $(T+S)^* = T^* + S^*$
   (ii) $(\alpha T)^* = \bar{\alpha} T^*$
   (iii) $(TS)^* = S^* T^*$
   (iv) $(T^*)^* = T$

4. (a) Define linear dependence and linear independence of vectors, (2+2+6=10)
   (b) Examine the linear dependence or independence of the set of vectors S={(1,1,1), (1,1,0),(1,0,0)} of $V_3(R)$.

5. (a) State the Gram Schmidt Orthogonalization theorem. (3+7=10)
   (b) Apply Gram Schmidt Orthogonalization process to the vectors $\beta_1 = (1,0,1), \beta_2 = (1,0,-1), \beta_1 = (0,3,4)$ to obtain an orthonormal basis
   for $V_3(R)$ with standard inner product.

6. (a) Let $M = \begin{pmatrix} -3 & 1 & 0 & & & & \\ 0 & -3 & 1 & & & & \\ 0 & 0 & -3 & & & & \\ & & & 5 & 1 & & \\ & & & 0 & 5 & & \\ & & & & & 5 & 1 \\ & & & & & 0 & 5 \end{pmatrix}$ (5+5=10)

Find the characteristic polynomial and minimal polynomial of M.

(b) Suppose the characteristic polynomial and minimal polynomial of an operator T are respectively

$$\Delta(t) = (t-2)^4(t-3)^3 \text{ and } m(t) = (t-2)^2(t-3)^2$$

Find all possible Jordan Cannonical form with these conditions.

7. (a) Define bilinear form of a vector space. (4+6=10)

(b) Let f be a bilinear form in R^2 defined by

$$f(x_1, x_2), (y_1, y_2) = x_1y_1 + x_2y_2$$

Find the matrix of f in each of the ordered basis $\{(1,-1),(1,1)\}$

8. (a) Define self adjoint operator, unitary operator and normal operator. (2+2+2+4=10)

(b) If U is a linear operator on an inner product space V(K), then show that the following conditions are equivalent

(i) $U^* = U^{-1}$

(ii) $UU^* = U^*U = I$

(iii) $< U(u), U(v) > = < u, v >$, for all $u, v \in V$

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[PART-A : Objective]

Choose the correct answer from the following: **1X20=20**

1. A non empty subset W of a vector space V(F) is subspace of V if for $\alpha, \beta \in F$ and $x, y \in W$
   a. $\alpha + \beta \in F$ b. $\alpha x + \beta y \in W$
   c. $\alpha x + \beta \in W$ d. $\alpha + \beta y \in W$

2. If V is a vector space over the field K and $x, y \in V$, $\alpha \in K$, then which of the following is false
   a. $\alpha\bar{0} = \bar{0}$
   b. $\alpha(-x) = -\alpha x$
   c. $x + y = x + z \Rightarrow x = z$
   d. $\alpha(x - y) = \alpha x - \alpha y$

3. The co-efficient of highest degree term of a monic polynomial is
   a. 0 b. 1 c. 2 d. 3

4. A linear operator T on an inner product space V(K) is called self adjoint if
   a. $< T(u), v > = < v, T(u) >$, for all $u, v \in V$
   b. $< T(u), v > = < u, T(v) >$, for all $u, v \in V$
   c. $< T(u), v > = < v, u >$, for all $u, v \in V$
   d. $< T(u), v > = < u, v >$, for all $u, v \in V$

5. The singleton set $\{v\}$ is linearly independent iff
   a. $v = 0$
   b. $v \neq 0$
   c. v is scalar
   d. None of these

6. Let V be a vector space over a field F and $u, v \in V$, $\alpha \in F$, then which of the following is true
   a. $|< u, v >| \geq \| u \| \| v \|$
   b. $|< u, v >| \leq \| u \| \| v \|$
   c. $|< u, v >| \geq \| u \| / \| v \|$
   d. $|< u, v >| \geq \| v \| / \| u \|$

7. Consider the following:
   (i) Let R be the field of real numbers and $W = \{(x, y, z) | x, y, z \in R\}$
   (ii) Let R be the field of real numbers and $W = \{(x, y, z)\} | x, y, z \in Q$, where Q is the set of rational numbers.
   Which of these is a vector space of $V_3(R)$?
   a. Only(i) b. Only(ii)
   c. Both (i) and (ii) d. Neither (i) nor (ii)

8. If $T: U \to V$ is a linear transformation, then the range of T is defined as
   a. $\{y \in V: T(x) = y: for\ some\ x \in U\}$
   b. $\{x \in U: T(x) = \bar{0}: for\ some\ \bar{0} \in V\}$
   c. $\{y \in V: T(x) = -x: for\ some\ x \in U\}$
   d. None of these

9. If $\lambda \neq 0$ is an eigen value of an invertible operator T , then the eigenvalue of T^{-1} is
   a. λ b. λ^{-1} c. 0 d. 1

10. If V is an inner product space, then for $u (\neq 0) \in V$,
    a. $\| u \| \leq 0$ b. $\| u \| \geq 0$
    c. $\| u \| = 0$ d. $\| u \| > 0$

11. The dimension of null space of a linear transformation T is called
    a. Rank(T) b. Nullity(T)
    c. Range(T) d. None of these

12. If A is a matrix of order n, then A is invertible if and only if
    a. $A \neq 0$ b. $A^{-1} = 0$
    c. $|A| \neq 0$ d. $|A| = 0$

13. Let $u = (x_1, x_2, x_3)$, $v = (y_1, y_2, y_3)$ and $f(u, v) = 3x_1y_1 - 2x_1y_2 + 5x_2y_1 + 7x_2y_2 - 8x_2y_3 + 4x_3y_2 - x_3y_3$, then the matrix representation A of f is
    a. $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 0 \end{pmatrix}$
    b. $A = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 0 & 0 \end{pmatrix}$
    c. $A = \begin{pmatrix} 3 & -2 & 0 \\ 5 & 7 & -8 \\ 0 & 4 & -1 \end{pmatrix}$
    d. None of these

14. If T is a linear operator on a finite dimensional vector space V and scalar c is an eigen value of T, then
    a. $\det(T - cI) \neq 0$
    b. $\det(T - cI) = 0$
    c. $\det(T - cI) > 0$
    d. $\det(T - cI) < 0$

15. Let V be an inner product space over a field F and $u \in V$, $\alpha \in F$. Which of the following is true

a. $\| \alpha u \| > |\alpha| \, \| u \|$
b. $\| \alpha u \| < |\alpha| \, \| u \|$
c. $\|\alpha u\| = |\alpha| \|u\|$
d. None of these

16. Let $A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$, then the characteristic polynomial for A is

a. $x^3 + 5x^2 + 8x + 4$
b. $x^3 + 5x$
c. $x^3 - 5x^2 + 8x - 4$
d. None of these

17. A linear operator on an inner product space V(K) is said to be unitary if and only if

a. $TT^* = T^*T$
b. $TT^* = T^{-1}$
c. $T^* = T^{-1}$
d. None of these

18. If $A = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}$, then the minimal polynomial of A is

a. $(t - \lambda)$
b. $(t - \lambda)^2$
c. $(t - \lambda)^3$
d. None of these

19. A bilinear form f on a vector space V(K) is said to be anti symmetric if for all $u, v \in V$______

a. $f(u, v) = -f(v, u)$
b. $f(u, v) = f(v, u)$
c. $f(u, v) = f(0, v)$
d. None of these

20. Let U be a linear operator on any inner product space V(K) and $\| U \| = \| u \|$, for all $u \in U$, then which of the following is correct.

a. $U^* = U$
b. $U^* = I$
c. $U^*U = I$
d. None of these

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

Question Paper CUM Answer Sheet

[PART (A) : OBJECTIVE]

Serial no. of the main Answer sheet

Course : ....

Semester : .... Roll No : ....

Enrollment No : .... Course code : ....

Course Title : ....

Session : 2016-17 Date : ....

Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- The student shall write the answer in the box where it is provided.
- The student shall not overwrite / erase any answer and no mark shall be given for such act.
- Hand over the question paper cum answer sheet (Objective) within the allotted time (20 minutes / 10 minutes) to the invigilator.

Full Marks	Marks Obtained	Remarks
20		

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*