M.Sc. MATHEMATICS
SECOND SEMESTER
REAL ANALYSIS & LEBESGUE MEASURE
MSM-201

[PART-A : Objective]

Choose the correct answer from the following: **1X20=20**

1. A sequence $\{f_n\}$ of functions is said to converge uniformly on X to a function f if for every $\varepsilon > 0$, we have
   a. $|f_n(x) + f(x)| > \varepsilon$
   b. $|f_n(x) + f(x)| < \varepsilon$
   c. $|f_n(x) - f(x)| < \varepsilon$
   d. None of these

2. Let (X, d) be a metric space and s=$\{s_n\}$ be a sequence in X. Then s is said to be a ……………………..in X iff for every $\varepsilon > 0$, there exists a positive integer m(ε) such that $p \geq m(\varepsilon) \,\&\, n \geq m(\varepsilon) \Rightarrow d(s_p, s_n) < \varepsilon$
   a. Monotonic sequence b. Cauchy sequence
   c. Convergent sequence d. Divergent sequence

3. A sequence $\{f_n\}$ of real valued functions defined on a metric space X is said to be uniformly bounded on X if
   a. $|f_n(x)| = M$
   b. $|f_n(x)| > M$
   c. $|f_n(x)| < M$
   d. $|f_n(x)| = 0$

4. By Weirestrass's M Test the given series $cosx + \frac{cos2x}{2^2} + \frac{cos3x}{3^2} + \cdots$ will
   a. diverge uniformly b. converge uniformly
   c. both of these d. None of these

5. Let $f_n(x) = x^{1/n}$ for $x \in [0,1]$. Then
   a. $\lim_{n\to\infty} f_n(x)$ exists for all x $\in [0,1]$.
   b. $\{f_n\}$ converges uniformly on [0,1]
   c. $\lim_{n\to\infty} f_n(x)$ defines a continuous function on x $\in [0,1]$.
   d. $\lim_{n\to\infty} f_n(x) = 0$ for all x $\in [0,1]$.

6. The series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
   a. Does not converge b. Converges to 1
   c. Converges to 2 d. None of these

7. The function $f(x) = \frac{1}{x}, x > 0$ is
   a. continuous but not uniformly continuous.
   b. uniformly continuous but not continuous.
   c. neither continuous nor uniformly continuous.
   d. discontinuous everywhere.

8. In properties of Lebesgue integral for bounded measurable function f on [a,b] , if A_k is a finite or infinite sequene of disjoint measurable subsets of [a,b] whose union A has finite measure, then
   a. $\int f = 0$ b. $\int f = \sum_k \int f$
   c. $\int f = 1$ d. $\int f = -\sum_k \int f$

9. If f is Lebesgue integrable , then
   a. $\left|\int_a^b f\right| \geq \int_a^b |f|$ b. $\left|\int_a^b f\right| \geq -\int_a^b |f|$
   c. $\int_a^b |f| \geq \left|\int_a^b f\right|$ d. $\left|\int_a^b f\right| < \int_a^b |f|$

10. Every upper Riemann integral is …………………..every upper Lebesgue integral.
    a. equal to b. greater than or equal to
    c. less than or equal to d. less than

11. If f is a function of bounded variation, then $1/f$ is also a function of bounded variation if for a positive integer k, ____________
    a. $|f(x)| \leq k$
    b. $|f(x)| \geq k$
    c. $|f(x)| = k$
    d. $|f(x)| < k$

12. Let $f(x) = sinx$, then the total variation of f in $[0, \pi/2]$ is
    a. 0 b. 1
    c. 2 d. 3

13. Which of the following statement is true?
    a. A bounded monotonic function is a function of bounded variation.
    b. If f is a function of bounded variation on $[a, b]$, then it is not a function of bounded variation on $[a, c]$ and $[c, b]$.
    c. The product of two functions of bounded variation is not necessarily a function of bounded variation.
    d. All of the above.

14. If f is R-S integrable in $[a, b]$, then which of the following is true
    a. $\int_a^b f d\alpha > \int_{\underline{a}}^b f d\alpha$
    b. $\int_a^b f d\alpha < \int_{\underline{a}}^b f d\alpha$
    c. $\int_a^b f d\alpha = \int_{\underline{a}}^b f d\alpha$
    d. None of these

15. $\int_0^2 x^2 dx^2$ will be equal to

a. 2
b. 4
c. 6
d. 8

16. If f is a function of bounded variation in $[a, b]$, then

a. $V(f; a, b) \geq |f(a) - f(b)|$ with respect to the partition $\{a, b\}$
b. $V(f; a, b) \leq |f(a) - f(b)|$ with respect to the partition $\{a, b\}$
c. $V(f; a, b) \geq |f(b) - f(a)|$ with respect to the partition $\{a, b\}$
d. $V(f; a, b) \leq |f(b) - f(a)|$ with respect to the partition $\{a, b\}$

17. If f is R-S integrable and α is monotonic increasing function on [a, b] such that α' is also R-S integrable, then which of the following holds

a. $\int_a^b f\,d\alpha = \int_a^b f\,d\alpha'$
b. $\int_a^b f\,d\alpha > \int_a^b f\,d\alpha'$
c. $\int_a^b f\,d\alpha < \int_a^b f\,d\alpha'$
d. None of these

18. If f is a function of bounded variation on $[a, b]$ and $x \in [a, b]$, then the total variation function $V(f; a, x)$ is a__________

a. Monotonic decreasing function
b. Monotonic increasing function
c. Step function
d. Constant function

19. Let P_1 and P_2 be two partitions of $[a, b]$ and P^* is their common refinement, then which of the following will hold

a. $P^* = P_1 \cap P_2$
b. $P^* = P_1 \cup P_2$
c. $P^* = P_1 + P_2$
d. $P^* = P_1 - P_2$

20. If f and α are bounded functions on $[a, b]$ and α is a monotonic increasing function on $[a, b]$, then

a. $\int_a^b f\,d\alpha = \inf U(P, f, \alpha)$
b. $\int_a^b f\,d\alpha = \sup L(P, f, \alpha)$
c. $\int_a^b f\,d\alpha = U(P, f, \alpha)$
d. $\int_a^b f\,d\alpha = L(P, f, \alpha)$

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

Question Paper CUM Answer Sheet

[PART (A) : OBJECTIVE]

Serial no. of the main Answer sheet

Course : ....

Semester : .... **Roll No :** ....

Enrollment No : .... **Course code :** ....

Course Title : ....

Session : 2016-17 **Date :** ....

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Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- The student shall write the answer in the box where it is provided.
- The student shall not overwrite / erase any answer and no mark shall be given for such act.
- Hand over the question paper cum answer sheet (Objective) within the allotted time (20 minutes / 10 minutes) to the invigilator.

Full Marks	Marks Obtained	Remarks
20		

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*