REV-00
MSM/39/44

2017/12

M. Sc. MATHEMATICS
FIRST SEMESTER
LINEAR ALGEBRA
MSM – 105

Duration: 3 Hrs. **Marks: 70**

Part : A (Objective) = 20
Part : B (Descriptive) = 50

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. 3+4+3=10

Also verify that it is satisfied by A. Determine A^{-1}.

2. Show that 5×2=10

(i) "A linear operator T on the finite dimensional vector space V is diagonisable if and only if $\exists$ a basis β of V consisting of eigen vectors of T."

(ii) "Every complex vector space is a real vector space but the converse is not true."

3. Show that the set $S = \{(1,0,0),\ (1,1,0),\ (1,1,1),\ (0,1,0)\}$ generates the vector space $V_3(R)$, but it is not a basis. 10

4. Prove that 5×2=10

(i) "If T and S are self adjoint operators on an inner product space V, then TS is self adjoint $\Leftrightarrow TS = ST$.

(ii) "A necessary and sufficient condition that a linear operator T on a complex inner product space V (unitary space) be sefl-adjoint is that $< T(\alpha),\alpha >$ be real for every α.

5. Find the eigen values and the corresponding eigen space for the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 10

6. State the Gram-Schmidt orthogonalization process .Apply the Gram-Schmidt process to the vectors $u_1 = (1,0,1)$ $u_2 = (1,0,-1)$ $u_3 = (0,3,4)$ to obtain an orthonormal basis for $R^3(R)$ with the standard inner product. 2+6=10

7. Reduce to canonical form and find the rank and the index of real quadraticform 10

$$q(x_1, x_2, x_3) = 2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$$

8. (i) Show that the union of two subspaces is a subspace if and only if one is contained in the other. 5+5=10

(ii) Show by an example that the union of two subspaces of a vector space $V(F)$ is not necessarily a subspace of $V(F)$.

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[PART-A : Objective]

Choose the correct answer from the following : 1×20=20

1. Let T be a linear operator on R^2 which is represented by $A=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then T has .....
   a. no eigen value in R
   b. one eigen value in R
   c. two eigen values in R
   d. None of these

2. A linear operator T on the finite dimensional vector space V is diagonisable if and only if $\exists$ a..... β of V consisting of eigen vectors of T
   a. non-zero set
   b. Set
   c. Basis
   d. None of these

3. The union of two subspaces....
   a. Is always a subspace
   b. May not be a subspace
   c. Is a null set
   d. None of these

4. Every vector space has atleast
   a. One subspace
   b. Two subspaces
   c. Three subspaces
   d. Four subspaces

5. Under what condition on the scalar a to the vector $(1,1,1)$ and $(1,a,a)$ forms a basis of $C^3(C)$.
   a. $a=0$
   b. $a=1$
   c. $a=-1$
   d. $a=\pm 1$

6. Minimal polynomial and characteristic polynomial have same roots, they ....
   a. are same
   b. may not be same
   c. always different
   d. none of these

7. A linear transformation $T:U \to V$ is non-singular if and only if T is
   a. One-one
   b. Onto
   c. T is a null space
   d. None of these

8. A linear transformation T on a finite dimensional vector space is invertable if and only if
   a. T is non-singular
   b. T is singular
   c. T is onto
   d. None of these

9. Every square matrix is ...... its characteristic polynomial.
   a. A zero of
   b. non-zero of
   c. equal to
   d. None of these

10. If $p(x)$ is a minimal polynomial, then no polynomial over F which annihilates T has
    a. equal degree than $p(x)$
    b. higher degree than $p(x)$
    c. smaller degree than $p(x)$
    d. None of these

11. A subset S of $V(F)$ is said to be a basis of $V(F)$ if
    a. S is linearly independent
    b. S is linearly dependent
    c. S is a null space
    d. None of these

12. If W_1 and W_2 be two subspaces of a finite dimensional vector space $V(F)$, then
    a. $\dim (W_1+W_2)=\dim W_1+\dim W_2$
    b. $\dim (W_1+W_2)=\dim W_1+\dim W_2+\dim (W_1\cap W_2)$
    c. $\dim (W_1+W_2)=\dim W_1+\dim W_2-\dim (W_1\cap W_2)$
    d. None of these

13. If $p(x)$ is a minimal polynomial, then $p(x)$ is ......
    a. a characteristic polynomial
    b. a monic polynomial
    c. Singular
    d. Non-singular

14. Suppose T is a linear operator on an inner product space. Then T is normal if and only if its real and imaginary parts ....

a. Are equal
b. Are zero
c. Commute
d. None of these

15. If $|A| = 0$, then the rank of A

a. Less than the number of variables and the system is linearly dependent
b. Less than the number of variables and the system is linearly independent
c. Greater than the number of variables and the system is linearly dependent
d. None of these

16. A subset W of a vector space $V(F)$ is a subspace of $V(F)$ if and only if

a. $v_1, v_2 \in W$ and $a, b \in F \Rightarrow av_1 + bv_2 \in W$
b. $0 \in W$
c. W is a null space
d. None of these

17. A complex inner product space is often referred to as a...

a. Unitary space
b. Euclidean space
c. Normal space
d. None of these

18. Trivial subspaces of V are

a. V itself
b. $\{0\}$
c. V itself and $\{0\}$
d. None of these

19. The intersection of arbitrary subspaces of a vector space is ....of that vector space.

a. May not be a subspace
b. A subspace
c. Normal subspace
d. Orthonormal subspace

20. If rank of the co-efficient matrix is same as the number of variables, then the system has

a. Non-zero solution
b. No solution
c. One solution
d. Zero solution

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

[PART (A) : OBJECTIVE]

Duration : 20 Minutes

Serial no. of the main Answer sheet

Course : ....

Semester : .... **Roll No :** ....

Enrollment No : .... **Course code :** ....

Course Title : ....

Session : 2017-18 **Date :** ....

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Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- Students shall tick (✓) the correct answer.
- No marks shall be given for overwrite / erasing.
- Students have to submit the Objective Part (Part-A) to the invigilator just after completion of the allotted time from the starting of examination.

Full Marks	Marks Obtained
20	

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*