REV-00
MSM/44/50

2017/12

M. Sc. MATHEMATICS
FIRST SEMESTER
ANALYSIS - I
MSM – 101

Duration: 3 Hrs. **Marks: 70**

Part : A (Objective) = 20
Part : B (Descriptive) = 50

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. State and prove the Cauchy's General Principle of convergence of sequence. 3+7=10

2. State the second mean value theorem. Show that $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$, if $0<u<v$ and deduce that $\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$ 2+8=10

3. (a) If $\lim_{x\to 0}\frac{a\sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of a and the limit. 7+3=10

   (b) Find $\lim_{x\to 1} x^{\frac{1}{1-x}}$

4. State and prove Taylor's theorem. 2+8=10

5. (a) Define uniform convergence of sequence of functions. Write the difference between pointwise convergence and uniform convergence. 2+3+5 = 10

   (b) Prove that the sequence $\{f_n\}$ where $f_n(x)=\frac{x}{1+nx^2}$, x is real converges uniformely on any interval I.

6. Test the convergence of the following series: 4+6=10

   (a) $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots\ldots$

   (b) $\sum \frac{3.6.9\ldots.3n}{7.10.13\ldots.(3n+4)}x^n$, $x>0$

7. Answer the following: 2+3+5 = 10

(a) Define compactness in a metric space.

(b) Is the open interval (0, 1) compact with the usual metric. Justify your answer.

(c) Prove that continuous image of a connected set is connected.

8. (a) Define continuity in a metric space. 3+7=10

(b) If (X,d_1) *and* (Y,d_2) are two metric spaces and $f:X \to Y$ .Then prove that f is continuous if and only if the inverse image under f of open subsets of Y is an open subset in X.

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[PART-A : Objective]

Choose the correct answer from the following: 1×20=20

1. The sequence $\left\{1+\frac{1}{n}\right\}$ converges to
   a. 0
   b. 1
   c. 2
   d. 3

2. A subset of a metric space (X, d) is closed iff
   a. D(A) $\subseteq A$
   b. A $\subseteq D(A)$
   c. D(A) $\not\subset A$
   d. None of these

3. If $\sum u_n$ is a positive term series such that $\lim_{n\to\infty}(u_n)^{1/n} = l$, then the series converges if
   a. $l > 1$
   b. $l < 1$
   c. $l = 1$
   d. None of these

4. Which of the following statement is false
   a. Every closed subset of a compact metric space is compact.
   b. The interval [1, 3] with the usual metric is compact.
   c. A subset A of a metric space is said to be connected if it can be expressed as the union of two non empty separated sets.
   d. Continuous image of a connected set is connected.

5. The Heine-Borel theorem states that
   a. Continuous image of a compact set is compact.
   b. Every closed and bounded subset of the real line is compact.
   c. Every compact subset of a metric space is closed.
   d. None of these

6. Every convergent sequence has
   a. A unique limit point
   b. Atleast one limit point
   c. Atmost two limit points
   d. None of these

7. Two sets A and B in a metric space (X, d) are said to be separable if
   a. $A \cap \overline{B} \neq \phi$ and $\overline{A} \cap B = \phi$
   b. $A \cap \overline{B} = \phi$ and $\overline{A} \cap B = \phi$
   c. $A \cap \overline{B} \neq \phi$ and $\overline{A} \cap B \neq \phi$
   d. None of these

8. Let (X, d) be a metric space. Then a mapping $f : X \to X$ is said to be a contraction mapping if there exists a positive real number α with $\alpha < 1$ such that for all $x, y \in X$
   a. $d(f(x), f(y)) = \alpha\, d(x, y)$
   b. $d(f(x), f(y)) \geq \alpha\, d(x, y)$
   c. $d(f(x), f(y)) \leq \alpha\, d(x, y)$
   d. None of these

9. A function $f : (X, d) \to (Y, d')$ is isometry if
   a. $d(x, y) \leq d'(f(x), f(y))$
   b. $d(x, y) \geq d'(f(x), f(y))$
   c. $d(x, y) = d'(f(x), f(y))$
   d. None of these

10. $f(x) = |x| + |x - 1, \forall x \in R|$ is continuous but not derivable at ...
   a. X=0, 1
   b. X=0
   c. X=1
   d. None of these

11. $\lim_{x\to 0} \frac{\tan x - x}{x - \sin x} = ?$
   a. 0
   b. 2
   c. 1
   d. None of these

12. Whether $f(x) = |x|$ is derivable or not at the origin ?
   a. Derivable
   b. Can not say
   c. Not derivable
   d. None of these

13. A function which is derivable at a point is necessarily ... at that point.
   a. Continuous
   b. Bounded
   c. Differentiable
   d. None of these

14. $\lim_{x\to\frac{\pi}{2}} \frac{\tan 5x}{\tan x} = ?$

a. 5 c. -1/5
b. 0 d. 1/5

15. If $\int_a^b f dx$ exists, this implies that the function f is .... And ..... over $[a,b]$.

a. Derivable, continuous c. Derivable, bounded
b. Bounded , integrable d. None of these

16. If two functions f and g defined on $[a,b]$ are continuous on $[a,b]$, derivable on $]a,b[$, $g'(x) \neq 0$ for any $x \in [a,b]$, then there exists a real number $c \in]a,b[$, such that

a. $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(x)}{g'(x)}$
b. $\frac{f(b)+f(a)}{g(b)+g(a)} = \frac{f'(x)}{g'(x)}$
c. $\frac{f(b)-f(a)}{g(b)-g(a)} = 0$
d. None of these

17. Let $\{f_n(x)\}$ be a sequence of function such that $\lim_{n\to\infty} f_n(x) = f(x),\ x \in [a,b]$ and $f_n \to f$ uniformly on $[a,b]$ if and only if $M_n \to 0$ as $n \to \infty$. What is M_n?

a. $M_n = f$ c. $M_n = \sup_{x\in[a,b]} |f_n(x) - f(x)|$
b. $M_n = \inf_{x\in[a,b]} |f_n(x) - f(x)|$ d. None of these

18. Uniform convergence of a sequence is dependent on
a. x c. on ε
b. *both x and* ε d. None of these

19. A sequence which is not pointwise convergent cannot be ....
a. Uniformly convergent c. Continuous
b. Convergent d. None of these

20. A conditional convergent series is
a. Absolutely convergent
b. Convergent but not absolutely convergent
c. Divergent
d. None of these

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

[PART (A) : OBJECTIVE]

Duration : 20 Minutes

Serial no. of the main Answer sheet

Course : ....

Semester : .... Roll No : ....

Enrollment No : .... Course code : ....

Course Title : ....

Session : 2017-18 Date : ....

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Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- Students shall tick (✓) the correct answer.
- No marks shall be given for overwrite / erasing.
- Students have to submit the Objective Part (Part-A) to the invigilator just after completion of the allotted time from the starting of examination.

Full Marks	Marks Obtained
20	

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*