

**M.Sc. MATHEMATICS
SECOND SEMESTER
TOPOLOGY & FUNCTIONAL ANALYSIS
MSM – 202**

[USE OMR FOR OBJECTIVE PART]

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

(Objective)

Marks: 20

Choose the correct answer from the following:

1×20=20

- In the discrete topological space $(X, \mathcal{D})$ where $X = \{p, q, r, s\}$ the number of open sets in X is?
 - 2
 - 8
 - 4
 - 16
- Let τ_f be cofinite topology on $\mathbb{N} = \{1, 2, 3, 4, \dots\}$. If $A \subset \mathbb{N}$ is open in $\mathbb{N}$ then A can be
 - $A = \{24, 25, 26, 27, 28, 29, \dots\}$
 - $A = \{1, 2, 5, 7, 11, 12, 13, 15\}$
 - $A = \{1, 3, 5, 7, 9, \dots\}$
 - $A = \{0, 2, 4, 6, 8, \dots\}$
- If $X = \{a, b, c\}$ then the number of topologies on X having exactly four members is
 - 7
 - 8
 - 9
 - 10
- Let X be an indiscrete topological space and A be a subset of X such that A neither an empty nor a singleton set. Then derived set of A is
 - ϕ
 - X
 - A
 - None of these
- Let $f: (X, T) \rightarrow (Y, T^*)$ be any function. f is continuous if
 - T and T^* are any topologies.
 - T is discrete topology and T^* is any topology
 - T is any topology and T^* is discrete topology.
 - None of the above is true.
- Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be bases for two topologies T_1 and T_2 respectively on a non-empty set X . Then T_1 is coarser than T_2 if
 - $1 \not\subset 2$
 - $2 \not\subset 1$
 - $1 \subset 2$
 - $2 \subset 1$
- Consider $\tau = \{\emptyset, X\}$ and the discrete topology on X . Then τ is
 - a base for
 - neither a base nor a subbase for
 - a subbase for
 - None of these

8. Find out the false statement of the following
- Every metric space is T_1 .
 - If $X = \{a, b\}$ and $T = \{X, \phi, \{a\}\}$ then (X, T) is a T_1 -space.
 - Every metric space is T_2 .
 - If $X = \{a, b, c\}$ and $T = \{X, \phi, \{a\}, \{b, c\}\}$ then (X, T) is a regular space.
9. Any discrete topological space is
- First countable
 - Both first countable and second countable
 - Second countable
 - Neither first countable nor second countable.
10. Point out the wrong statement of the following:
- Any subspace of a first countable space is first countable.
 - The real line $\mathbb{R}$ with discrete topological is second countable.
 - Any second countable space is also first countable.
 - The real line $\mathbb{R}$ with usual topological is second countable.
11. A constant function $f: (X, T) \rightarrow (X, T^*)$ is continuous if
- T and T^* are any topologies on X
 - T is coarser than T^*
 - T is finer than T^*
 - All the above are true.
12. Let $X = \{a, b, c\}$ and $T = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Which of the following statement is true?
- X is a normal space
 - X is a T_1 -space.
 - X is a regular space.
 - X is T_3 -space
13. Find out the true statement from the following statements:
A fraction in $[0, 1]$ is called dyadic if it can be expressed as
- $\frac{m}{2^n}, m, n \in \mathbb{Z}, n$ positive.
 - $\frac{m}{2^n}, m, n \in \mathbb{Z}, m$ is positive.
 - $\frac{m}{2^n}, m, n \in \mathbb{Z}, m, n$ positive, $m \leq 2^n$
 - $\frac{m}{3^n}, m, n \in \mathbb{Z}, m, n$ positive.
14. Let $D = \{\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{16}, \frac{3}{16}, \dots\}$ be set of dyadic fractions in $[0, 1]$. Then choose the correct statement of the following
- $\bar{D} \subset [0, 1]$
 - $D = \bar{D}$
 - $\bar{D} = [0, 1]$
 - $\bar{D} \subset D$
15. Let $\mathbb{N}$ be the set of natural numbers and let $T_1 = \{\phi, \mathbb{N}, \{5, 11\}, \{5\}, \{11\}\}$ and $T_2 = \mathcal{P}(\mathbb{N})$, where $\mathcal{P}(\mathbb{N})$ is the power set of $\mathbb{N}$. Then which of the following is/are true?
- T_1 is compact but T_2 is not compact.
 - Both T_1 and T_2 are compact.
 - T_2 is compact but T_1 is not compact.
 - Neither T_1 nor T_2 is compact.

16. Which of the following is/are true?
- a. In discrete topological space, every open cover has a finite subcover.
 - b. In indiscrete topology, every open cover has a finite subcover.
 - c. In $\mathbb{R}$ with usual topology, every open cover has a finite subcover.
 - d. None of these
17. Which of the following is/are not always true?
- a. The discrete topological space is compact.
 - b. All finite topological spaces are compact.
 - c. The indiscrete topological space is compact.
 - d. All the above.
18. Which of the following is/are true?
- a. $\mathcal{A} = \left\{ \left(a, \frac{1}{2} \right) : a = (m, n) \in \mathbb{Z} \times \mathbb{Z} \right\}$ is an open cover on $\mathbb{R}^2$ with usual topology.
 - b. $\mathcal{B} = \left\{ \left(\frac{1}{n}, 1 \right] : n \in \mathbb{Z}^+ \right\}$ is not an open cover on $(0, 1]$ with usual topology.
 - c. $\mathcal{C} = \{(n, n + 2) : n \in \mathbb{Z}\}$ is an open cover on $\mathbb{R}$ with usual topology.
 - d. None of these
19. Which of the following is/are open cover of discrete topological space $(X, \mathcal{D})$, where $X = \{a, b, c\}$?
- a. $\{\{a\}, \{b\}, \{c\}\}$
 - b. $\{\{a\}, \{b\}, \{c\}, \{a, b\}\}$
 - c. $\{\phi, X, \{a\}\}$
 - d. All the above
20. Let $\mathfrak{I} = \{\phi, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology on $X = \{a, b, c, d, e\}$ and let $A = \{a, d, e\}$. Then $\mathfrak{I}_A$ is equal to
- a. $\{\phi, A, \{a\}, \{d\}, \{a, d\}, \{d, e\}\}$
 - b. $\{\phi, A, \{a\}, \{d\}, \{c, d\}, \{a, e\}\}$
 - c. $\{\phi, A, \{a\}, \{d\}, \{a, d\}, \{c, d\}\}$
 - d. None of these
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(Descriptive)

Time : 2 hrs. 30 mins.

Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Define a topological space (X, T) . Consider $X = \{a, b, c, d, e\}$ and collections of subsets of X as given below: 5+5=10
- $$T_1 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$$
- $$T_2 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$
- $$T_3 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{a, b, d, e\}\}$$
- Examine if T_1, T_2, T_3 are topologies on X or not.
- b. Let T be a topology on a set X consisting of four sets i.e., $T = \{\phi, X, A, B\}$ where A and B are non-empty distinct proper subsets of X . Find the conditions to be satisfied by A and B .
2. a. What are open and closed sets in a topological space (X, T) ? 5+5=10
If $X = \{p, q, r, s, t\}$ and $T = \{X, \phi, \{p\}, \{r, s\}, \{p, r, s\}, \{q, r, s, t\}\}$ is a topology on X , then find all the open and closed subsets of X .
- b. Define interior and closure point of a set A in a topological space (X, T) and hence explain what you mean by A^0 (or $\text{int}(A)$) and $\bar{A}$ for the set $A \subset X$. In reference to the topological space (X, T) as given in the question 2(a) above if $A = \{q, r, s\}$, then find A^0 and $\bar{A}$.
3. a. Define a base for a topological space (X, T) . Prove that $\mathcal{B}$ will be a base for some topology on X if and only if 5+5=10
- (i) $X = \bigcup \{B : B \in \mathcal{B}\}$ and
- (ii) For any $B_1, B_2 \in \mathcal{B}$, $B_1 \cap B_2$ is the union of members of $\mathcal{B}$ i.e., for $p \in B_1 \cap B_2$ there exists $B_p \in \mathcal{B}$ such that $p \in B_p \subset B_1 \cap B_2$.
- b. Explain how any class of subsets of a non-empty set X can generate a unique topology on X . Consider $X = \{a, b, c, d\}$ and the class of subsets $\mathcal{A} = \{\{a, b\}, \{b, c\}, \{d\}\}$. Find the topology T generated by $\mathcal{A}$ on X .

4. a. Let (X, T) and (Y, T^*) be two topological spaces and $f: X \rightarrow Y$ be any function. What do you mean by continuity of f from A to B ? 5+5=10
 Consider $X = \{a, b, c, d\}, T = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$
 $Y = \{x, y, z, w\}, T^* = \{\phi, Y, \{x\}, \{y\}, \{x, y\}, \{y, z, w\}\}$
 Define $f: X \rightarrow Y$ with $f = \{(a, y), (b, z), (c, w), (d, z)\}$ and
 $g: X \rightarrow Y$ with $g = \{(a, x), (b, x), (c, z), (d, w)\}$.
 Examine if f and g are continuous or not.
- b. Prove that a function $f: X \rightarrow Y$ is continuous if and only if the inverse image of every closed subset of Y is a closed subset of X .
5. a. Let $f: (X, T) \rightarrow (Y, T^*)$ be any function and $\mathcal{S}$ be a subbase for the topology T^* on Y . Prove that f is continuous if and only if the inverse of every member of the subbase $\mathcal{S}$ is an open subset of X i.e., $f^{-1}[S] \in T$ for every $S \in \mathcal{S}$. 5+5=10
- b. What are first countable and second countable topological spaces. Give example of each of them. Show that every second countable topological space is also first countable. Cite example to show that a first countable space is not necessarily second countable.
6. a. Let A be any subset of a second countable topological space X . If $\mathcal{G}$ is an open cover of A , then prove that $\mathcal{G}$ is reducible to a countable cover. 5+5=10
- b. If X satisfies second axiom of countability then prove that X is separable i.e., X has a countable dense subset.
7. a. Prove that a topological space X is T_1 if and only if every singleton set in X is closed. 5+5=10
- b. Prove that a topological space (X, T) is normal if and only if for every closed set F and open set H containing F there exists an open set G such that $F \subset G \subset \bar{G} \subset H$.
8. a. Prove that - The interval $(0, 1)$ with usual topology is not compact. 5+5=10
- b. Prove that - Every co-finite space are compact.

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