

M.Sc. MATHEMATICS
FOURTH SEMESTER
GRAPH THEORY
MSM – 401

**SET
A**

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs.

Full Marks: 70

(Objective)

Time: 30 min.

Marks: 20

Choose the correct answer from the following:

1×20=20

1. What do mean by pendent vertex.
a. Having degree 0
b. Having degree 1
c. Having degree 2
d. None
2. A graph is said to be isomorphic if
a. The same numbers of vertices
b. The equal numbers vertices
c. The equal numbers of edges
d. All of the above
3. What is the number of edges present in complete graph K_n having n vertices?
a. $\frac{n(n+1)}{2}$
b. $\frac{n(n-1)}{2}$
c. n^2
d. None of these
4. What is the maximum number of edges in a bipartite graph having 12 vertices.
a. 24
b. 38
c. 36
d. 32
5. A Graph contains m edges can be decomposed in
a. $2^m - 1$ ways
b. $2^{m-1} - 1$ ways
c. 2^m ways
d. None
6. What is the regularity of K_n .
a. n
b. $n - 1$
c. 0
d. 1
7. Conditions of planarity for n numbers of vertices and e numbers of edges
a. $e < 2n - 6$
b. $e < 2n - 6$
c. $e \leq 3n - 6$
d. $e = 2n - 6$

8. Which of the following is true for any simple connected graph with more than 2 vertices.
- No two vertices have same degree
 - At least two vertices have same degree
 - At least 3 vertices have same degree
 - All vertices have same degree
9. A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graph of n vertices, n is
- A multiple of 4
 - Even
 - Odd
 - Congruent to $0 \pmod 4$, or $1 \pmod 4$
10. A vertex which is not adjacent to every other vertex is called ___ vertex.
- Isolated
 - Pendant
 - Incident
 - Simple
11. For a given graph G having v vertices and e edges which is connected and has no cycles, which of the following statements is true?
- $v = e$
 - $v = e + 1$
 - $v + 1 = e$
 - $v = e - 1$
12. A graph is if it has at least one pair of vertices without a path between them.
- Complete
 - Connected
 - Disconnected
 - Trivial
13. Which of the following statements is/are TRUE for Tree?
 P: Every two points of G are joined by a unique path.
 Q: G is disconnected and
- $$p = q - 1$$
- P only
 - Q only
 - Both P and Q
 - Neither P and Q
14. Let $\kappa(G)$ = vertex connectivity, $\lambda(G)$ = edge connectivity and $\delta(G)$ = minimum degree of graph G . Then
- $\kappa(G) \leq \delta(G) \leq \lambda(G)$
 - $\lambda(G) \leq \delta(G) \leq \kappa(G)$
 - $\delta(G) \leq \kappa(G) \leq \lambda(G)$
 - $\kappa(G) \leq \delta(G) < \lambda(G)$
15. Number of edges incident with the vertex V is called?
- Degree of a graph
 - Handshaking lemma
 - Degree of a vertex
 - None of the above
16. The number of elements in the adjacency matrix of a graph having 7 vertices is
- 7
 - 14
 - 36
 - 49

17. If a graph is planar, then it is embeddable on a
- | | |
|-------------|-----------|
| a. Circle | b. Square |
| c. Triangle | d. Plane |
18. Total number of edges of a complete graph $K_{m,n}$
- | | |
|------------|------------------|
| a. $m + n$ | b. $m - n$ |
| c. mn | d. None of these |
19. Let G be a Null graph. Then
- | | |
|---------------------------|--------------------------|
| a. $V(G) = \phi$ | b. $E(G) = \phi$ |
| c. $ E(G) = \text{Even}$ | d. $ E(G) = \text{Odd}$ |
20. The minimum degree (δ) of the Petersen graph
- | | |
|------|------|
| a. 1 | b. 2 |
| c. 3 | d. 4 |

-- --- --

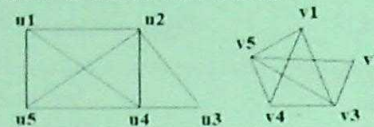
(Descriptive)

Time : 2 hrs. 30 mins.

Marks : 50

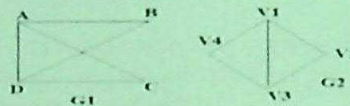
/ Answer question no.1 & any four (4) from the rest /

1. a. Determine whether the following pairs of graphs are isomorphic. Exhibit the isomorphism explicitly or prove that it does not exist. 5+5=10



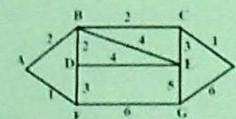
- b. Calculate the number of vertices and number of edges of the graph $G_1 (p_1, q_1)$ and $G_2 (p_2, q_2)$ graph by the rule of $G_1 + G_2, G_1 \cup G_2$

2. a. Establish the isomorphism of the two graphs given below by considering their adjacency matrices 5+5=10

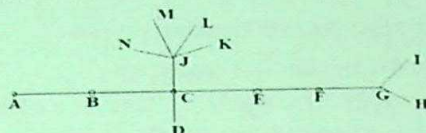


- b. Prove that the maximum number of edges among all p point graph with no triangle is $\left\lfloor \frac{p^2}{4} \right\rfloor$, ($\lfloor x \rfloor$ denotes the greatest integer not exceeding the real number x)

3. a. Explain the Dijkstra's algorithm to find the shortest path between the vertices A and H in the weighted graph given below 8+2=10



- b. Find the eccentricity, center, radius and diameter of each vertex from the following graph



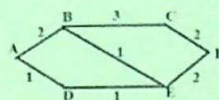
4. Let V be a point of a connected graph G then prove that the following statements are equivalent

10

- v is a cut point of G
- There exist a partition $V - \{v\}$ into subset U and W such that for each $u \in U$ the point v is on every $v - w$ path.
- There exist two points u and w distinct points from v such that v is on every $u - w$ path.

5. a. Find the shortest distance matrix and the corresponding shortest path matrix for all the pairs of vertices in the undirected graph given below using Warshall's algorithm

5+5=10



- b. Prove that a line x of a connected graph G is a bridge iff x is not on any cycle of G .

- Find the values of prefix expression $+, -, *, 2, 3, 5, /$, $\uparrow 2, 3, 8$
- Find the values of postfix expression $7, 2, -, 3, +, 2, 3, 2, +, -, 1, 3, -, *, /$

5+5=10

- Represent the expression $((a - c) * d) / (a + (b - d))$ as a binary tree and write the pre and postfix form of the expressions.
- In a connected plane (p, q) graph ($p \geq 3$) with r faces then show that $q \geq \frac{3r}{2}$ and $q \leq 3p - 6$

5+5=10

8.

- a. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$ and also prove that any self-complementary graph has $4n$ or $4n + 1$ vertices.
- b. If G is a connected plane graph having V , E , and F as the set of vertices, edges and faces respectively then prove that $|V| - |E| + |F| = 2$

5+5=10

== *** ==