

M.SC. MATHEMATICS
THIRD SEMESTER
MATHEMATICAL METHODS
MSM – 301
[USE OMR FOR OBJECTIVE PART]

2025/11

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

(Objective)

Marks: 20

1X20=20

Choose the correct answer from the following:

1. The values of λ for which determinant $\Delta = 0$ are known as
 - a. Eigen values
 - b. Eigen functions
 - c. Kernels
 - d. Separable kernels
2. The solution of Volterra integral equation $y(x) = 1 + \int_0^x y(t) dt$ is
 - a. $y = e^x$
 - b. $y = e^{-x}$
 - c. $y = \frac{e^x}{2!}$
 - d. $y = 1 + e^x$
3. The resolvent kernel of integral equation with kernel $k(x, t) = 2x, (\lambda = 1)$ is
 - a. $2e^{x^2-t^2}$
 - b. $2xe^{x^2-t^2}$
 - c. $2xe^{x-t}$
 - d. $x-t$
4. The integral equation $y(x) = 1 + \frac{1}{\pi} \int_0^{2\pi} \sin(x+t) y(t) dt$ has
 - a. Infinite solution
 - b. No solution
 - c. Unique solution
 - d. None
5. A Laplace Transform exists when
 - a. Function is piece-wise continuous
 - b. Function is of exponential order
 - c. Both a and b
 - d. None
6. If $\frac{dy}{dx} + ay = e^{-bt}$ with initial condition $y(0)=0$. Then Laplace transform $y(s)$ of the solution $y(t)$ is
 - a. $\frac{1}{(s+a)(s+b)}$
 - b. $\frac{1}{b(s+a)}$

c. $\frac{1}{a(s+b)}$

d. None

7. If the Laplace transform of function $f(t)$ is given by $\frac{s+3}{(s+1)(s+2)}$, then $f(0)$ is

a. $\frac{3}{2}$

b. $\frac{1}{2}$

c. 0

d. 1

8. Laplace transform of t is

a. $\frac{1}{s}$

b. $\frac{1}{s^2}$

c. $\frac{1}{s^3}$

d. None

9. The value of α for which the integral equation $u(x) = \alpha \int_0^1 e^{x-t} u(t) dt$ has trivial solution

a. -2

b. -1

c. 1

d. 2

10. The resolvent kernel of Volterra's integral equation

$$y(x) = f(x) + \lambda \int_0^x k(x,t) y(t) dt \text{ with kernel } k(x,t) = 1 \text{ is}$$

a. $e^{\lambda(x-t)}$

b. $e^{\lambda(x+t)}$

c. -1

d. 1

11. What is the fourier transform of e^{ax} ($a > 0$)?

a. $\frac{p}{a^2 + p^2}$

b. $2 \frac{a}{a^2 + p^2}$

c. $-2 \frac{a}{a^2 + p^2}$

d. Cannot be found

12. What is the fourier sine transform of e^{-ax} ?

a. $\frac{4}{4 + p^2}$

b. $4 \frac{a}{4a^2 + p^2}$

c. $\frac{p}{a^2 + p^2}$

d. $2 \frac{p}{a^2 + p^2}$

13. The inverse Laplace transforms of $H(s) = \frac{s+3}{s^2+2s+1}$ for $t \geq 0$ is
- $3te^{-t} + e^{-t}$
 - $3e^{-t}$
 - $2te^{-t} + e^{-t}$
 - $4te^{-t} + e^{-t}$
14. The solution of integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}} dt = 1$ is
- $y(x) = \frac{1}{\pi\sqrt{x}}$
 - $y(x) = \frac{1}{\sqrt{\pi}}$
 - $y(x) = \frac{\pi}{\sqrt{x}}$
 - $y(x) = \frac{\pi}{\sqrt{x^3}}$
15. The convolution process associated with the Laplace transform in time domain results
- Simple multiplication in complex frequency domain
 - Simple division in complex frequency domain
 - Simple multiplication in complex time domain
 - Simple division in complex time domain
16. The Inverse Laplace transform of $\frac{1}{s+1}$ is
- e^{-t}
 - e^t
 - 1
 - e^1
17. Laplace transform of $3t^4$ is
- $\frac{72}{s^5}$
 - $\frac{24}{s^4}$
 - $\frac{18}{s^4}$
 - $\frac{12}{s^5}$
18. Given kernel $k(x, t) = 1; a = 0; b = 1$ then
- $D(\lambda) = \lambda$
 - $D(x, t; \lambda) = 1$
 - $D(\lambda) = \frac{\lambda}{3}$
 - $D(x, t; \lambda) = 0$

19.

An integral equation $\phi(x) = F(x) + \lambda \int_0^1 k(x,t)\phi(t) dt$ is homogeneous if

- a. $F(x) \neq 0$
- b. $F(x) = 0$
- c. $F(x) = 1$
- d. None

20.

The inverse laplace of $\frac{1}{s}$ is

- a. 1
- b. -1
- c. 0
- d. None

-- -- --

(Descriptive)

Time : 2 hrs. 40 min.

Marks : 50

[Answer question no.1 & any four (4) from the rest]

1.

Solve the integral equation $\psi(x) = \sin x - \frac{x}{4} + \frac{1}{4} \int_0^{\pi/2} xt\psi(t) dt$.

10

2. Considering the complex form of Fourier integral of

5+5=10

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{i\lambda(t-x)} dtd\lambda$$

- a. Explain how you would define the Fourier transform $F(s)$ and the inverse Fourier Transform $F(s)$. If $F(s)$ is the Fourier transformation of a function $f(x)$ then show that the Fourier Transform of

(i) the function $f(as)$ is $\frac{1}{a} F\left(\frac{s}{a}\right)$

(ii) the function $f(x-a)$ is $e^{isa} F(s)$

(b) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate

$$\int_0^{\infty} \frac{\sin x}{x} dx.$$

3. Solve by using Laplace transform technique

5+5=10

(a) $\frac{d^3 y}{dt^3} + 2 \frac{d^2 y}{dt^2} - \frac{dy}{dt} - 2y = 0$; $y(0) = 1, y'(0) = y''(0) = 2$.

(b) Evaluate $L^{-1} \left\{ \frac{1}{(s^2 + 4)(s+1)^2} \right\}$

4. Show that the resolvent kernel of the integral equation

10

$\phi(x) = 1 + \lambda \int_0^x xt \phi(t) dt$ is $xe^{\frac{\lambda}{3}(x^3 - t^3)}$ and hence solve the equation.

5. Solve the integral equation

10

$\phi(x) = 1 + x + \int_0^x (x-t) \phi(t) dt$; $\phi_0(x) = 1$

6. Transform the boundary value problem into Fredholm integral equation

10

$\frac{d^2 y}{dx^2} + \lambda y = 0$; $y(0) = y(l) = 0$.

7. Solve the integral equation $\eta(x) = \lambda \int_{-1}^1 (5xt^3 + 4x^2t + 3xt) \eta(t) dt$

10