

**M.Sc. MATHEMATICS
FOURTH SEMESTER
PARTIAL DIFFERENTIAL EQUATION-II
MSM – 402
[USE OMR FOR OBJECTIVE PART]**

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

Marks: 20

(Objective)

$1 \times 20 = 20$

Choose the correct answer from the following:

1. The Charpit's Auxillary equations of $zpq = p + q$ are
 - a. $\frac{dp}{p^2q} = \frac{dq}{pq^2} = \frac{dz}{-p(zq-1)-q(zp-1)} = \frac{dx}{1-zq} = \frac{dy}{1-zp}$
 - b. $\frac{dp}{p^2q} = \frac{dq}{pq^2} = \frac{dz}{p(zq-1)-q(zp-1)} = \frac{dx}{1-zq} = \frac{dy}{1-zp}$
 - c. $\frac{dp}{p^2q} = \frac{dq}{pq^2} = \frac{dz}{-p(zq-1)-q(zp-1)} = \frac{dx}{1+zq} = \frac{dy}{1-zp}$
 - d. None of the above
2. A Partial Differential Equation of second order is hyperbolic if
 - a. $S^2 - 4RT < 0$
 - b. $S^2 - 4RT > 0$
 - c. $S^2 - 4RT = 0$
 - d. $S^2 - 4RT \geq 0$
3. Laplace Equation in Spherical coordinate is
 - a. $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} = 0$
 - b. $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} = 0$
 - c. $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} = 0$
 - d. $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} = 0$
4. The characteristics of the equation $y^2 r - x^2 t = 0$ are
 - a. $x^2 + y^2 = c_1, x^2 - y^2 = c_2, c_1, c_2$ are arbitrary constants
 - b. $x^2 + y^2 = c_1, x - y = c_2, c_1, c_2$ are arbitrary constants
 - c. $x + y = c_1, x^2 - y^2 = c_2, c_1, c_2$ are arbitrary constants
 - d. $2x^2 + 2y^2 = c_1, x^2 - y^2 = c_2, c_1, c_2$ are arbitrary constants
5. Complete solution of $(D^2 - 3aDD' + 2a^2D'^2)z = 0$ is
 - a. $z = \phi_1(y + ax) + \phi_2(y + 2ax), \phi_1, \phi_2$ are arbitrary functions.
 - b. $z = \phi_1(y - ax) + \phi_2(y - 2ax), \phi_1, \phi_2$ are arbitrary functions.
 - c. $z = \phi_1(y - ax) + \phi_2(y + 2ax), \phi_1, \phi_2$ are arbitrary functions.

- d. none of the above
6. Monge's subsidiary equations are
 $Rdpdy + Tdqdx - Vdxdy = 0$
 a. $R(dy)^2 - Sdxdy + T(dx)^2 = 0$
 $Rdpdy + Tdqdx - Vdxdy = 0$
 b. $R(dy)^2 - Sdxdy - T(dx)^2 = 0$
 $Rdpdy - Tdqdx - Vdxdy = 0$
 c. $R(dy)^2 - Sdxdy + T(dx)^2 = 0$
 d. none of the above
7. The Value of S & V for the differential equation
 $(1+q)^2 r - 2(1+p+q+pq)s + (1+p)^2 t = 0$ is
 a. $-(1+p+q+pq), 0$ b. $(1+p+q+pq), 0$
 c. $(1+q)^2, 0$ d. none of the above
8. The partial differential equation $u_{xx} + u_{yy} = u_{zz}$ is
 a. parabolic b. Hyperbolic
 c. elliptic d. circle
9. The general form of homogeneous Linear Partial Differential equation with constant coefficient is known as
 a. $(A_1 D^2 + A_2 D^{-1} D^2 + \dots + A_n D^{-n})z = f(x, y)$ b. $(A_1 D^2 + A_2 D^{-1} D + \dots + A_n D^{-n})z = f(x, y, z)$
 c. $(A_1 D^2 + A_2 D^{-1} D + \dots + A_n D^{-n})z = f(x, y)$ d. $(A_1 D^2 + A_2 D^{-1} D + \dots + A_n D^{-n})z = f(x, y)$
10. The characteristics equation of $yr + (x+y)s + xt = 0$ are
 a. $\frac{dy}{dx} - \frac{y}{x} = 0$ & $\frac{dy}{dx} - \frac{x}{y} = 0$ b. $\frac{dy}{dx} + 1 = 0$ & $\frac{dy}{dx} - \frac{x}{y} = 0$
 c. $\frac{dy}{dx} - 1 = 0$ & $\frac{dy}{dx} - \frac{x}{y} = 0$ d. $\frac{dy}{dx} - 1 = 0$ & $\frac{dy}{dx} + \frac{x}{y} = 0$
11. Two dimensional Heat Equation is
 a. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k} \frac{\partial^2 u}{\partial t^2}$ b. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k} \frac{\partial u}{\partial t}$
 c. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k^2} \frac{\partial^2 u}{\partial t^2}$ d. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k^2} \frac{\partial u}{\partial t}$
12. For the derivation of one dimensional wave equation we consider
 a. A String b. A Membrane
 c. A Rod d. none of the above

13. Lagrange's Auxillary Equations are

- a. $Pdx + Qdy + Rdz = 0$ b. $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$
 c. $Pdx - Qdy - Rdz = 0$ d. $\frac{dx}{P} = -\frac{dy}{Q} = \frac{dz}{R}$

14. If $R = 0, S = 3, T = 0, U = 1, V = 2$, then the differential equation is

- a. $3s + (rt - s^2) = -2$ b. $3s + (rt + s^2) = 2$
 c. $3s + (rt - s^2) = 2$ d. $3s - (rt - s^2) = 2$

15. The general solution of One dimensional wave equation is

- a. $u(x, t) = X(x)T(t)$
 b. $u(x, t) = (c_1 t + c_2)(c_3 x + c_4)$
 c. $u(x, t) = (c_1 e^{cpt} + c_2 e^{-cpt})(c_3 e^{px} + c_4 e^{-px})$
 d. $u(x, t) = (c_1 \cos t + c_2 \sin t)(c_3 \cos x + c_4 \sin x)$

16. Complete integral of $q = 3p^2$ is

- a. $z = ax + 3ay + b$ b. $z = ax - 3a^2 y + b$
 c. $z = ax + 3a^2 y + b$ d. $z = ax + 3a^2 y$

17. Complete solution of $(D^3 - 4D^2 D' + 4DD'^2) = 0$ is

- a. $z = \phi_1(y) + \phi_2(y + 2x) + x\phi_3(y + 2x)$, ϕ_1, ϕ_2, ϕ_3 are arb constant
 b. $z = \phi_1(y) + \phi_2(y + 2x) + x\phi_3(y + 2x)$, ϕ_1, ϕ_2, ϕ_3 are arb constant
 c. $z = \phi_1(y) + \phi_2(y + 2x) + x\phi_3(y + 2x)$, ϕ_1, ϕ_2, ϕ_3 are arb constant
 d. $z = \phi_1(y) + \phi_2(y + 2x) + x\phi_3(y + 2x)$, ϕ_1, ϕ_2, ϕ_3 are arb constant

18. Which one of the following partial differential equation is elliptic type

- a. $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ b. $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$
 c. $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ d. $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$

19. The problem in which for the different value of the variable of x , the value of the function y and its derivative are given at a point known as

- a. Initial Value Problem
- b. Boundary value problem
- c. Laplace equation
- d. Fourier equation

20.

We know one-dimensional wave equation is $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, where $c^2 = ?$

- a. $\frac{T^2}{\rho}$
- b. $\frac{T}{\rho^2}$
- c. $\frac{T}{\rho}$
- d. None of the above

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(Descriptive)

Time : 2 hrs. 30 mins.

Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Solve

10

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \text{ where } c^2 = \frac{T}{m}$$

a constant, subject to condition which is boundary condition

$$u(0, t) = u(l, t) = 0$$

and initial condition

$$u(x, 0) = f(x), \frac{\partial u}{\partial t} = 0 \text{ at } t = 0,$$

where l is the length of the string.

2. Solve

5+5=10

a. $xy p + y^2 q = zxy - 2x^2$

b. $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$

3. Find the Singular and General Solution of $4(1+z^3) = 9z^4 pq$ 10

4. Prove that $\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$ 10

5. Reduce the following partial differential equation to canonical form 5+5=10

a. $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$

b. $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$

6. Solve 5+5=10

a. $xyp + y^2 q = zxy - 2x^2$

b. $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$

7. Solve by Monge's Method 5+5=10

(a) $(q+1)s = (p+1)t$

(b) $pq = x(ps - qr)$

8. Solve by Charpit's Method 5+5=10

(a) $z^2(p^2 z^2 + q^2) = 1$

(b) $z = px + qy + p^2 + q^2$

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