

**M.Sc. MATHEMATICS  
SECOND SEMESTER  
COMPLEX ANALYSIS  
MSM – 201**  
(USE OMR FOR OBJECTIVE PART)

**SET  
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

( Objective )

Marks: 20

1×20=20

*Choose the correct answer from the following:*

1. A continuous arc without multiple points is called
  - a. Jordan curve
  - b. Continuous arc
  - c. Contour
  - d. Rectifiable arc
2. A continuous function  $f(z)$  over a continuous rectifiable curve  $C$  is
  - a. Differentiable
  - b. integrable
  - c. Meromorphic
  - d. None
3. The value of  $\int_C z dz$ , where  $C$  is any curve joining the points  $z = a$  to  $z = b$ 
  - a.  $b - a$
  - b.  $b^2 - a^2$
  - c.  $\frac{1}{2}(b^2 - a^2)$
  - d. None
4. If  $C$  is given by the equation  $|z - a| = R$ , then the value of  $\int_C \frac{dz}{z - a}$  is
  - a.  $2\pi i$
  - b.  $\pi i$
  - c.  $\frac{\pi i}{2}$
  - d. None of the above
5. If  $f(z)$  is analytic and uniformly bounded in every domain, then
  - a.  $f(z)$  is zero
  - b.  $f(z)$  is constant
  - c.  $f(z)$  is discontinuous
  - d. None
6.  $f(z) = \bar{z}$  is
  - a. continuous for every  $z$ , not differentiable for any  $z$
  - b. continuous for some values of  $z$ , differentiable for every  $z$
  - c. discontinuous for every  $z$ , differentiable for every  $z$
  - d. neither continuous nor differentiable

7. Functions satisfying Laplace's equation are known as  
 a. Regular  
 b. Homomorphic  
 c. Harmonic  
 d. Conjugate
8. An analytic function with constant modulus  
 a. is variable  
 b. is constant  
 c. is zero  
 d. Does not exist
9. If only one value of function corresponds to each value of complex number is called  
 a. single valued  
 b. multiple valued  
 c. both a and b  
 d. None of the above
10. The value of  $\lim_{z \rightarrow \infty} \left( e^{-z} + \frac{1}{z} \right)$  is  
 a. 1  
 b. 0  
 c. -1  
 d. None of the above
11. A function which satisfies the Laplace equation is called  
 a. Harmonic  
 b. Conjugate  
 c. Unique  
 d. None of the above
12. The radius of convergence of the power series  $f(z) = \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$  is  
 a. 2  
 b. 4  
 c. -2  
 d. -4
13. If  $\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = L$ , then  $\sum u_n$  converges if  
 a.  $L > 1$   
 b.  $L < 1$   
 c.  $L = 1$   
 d. None
14. The singular point  $z = 0$  of  $f(z) = \frac{\sin z}{z}$  is  
 a. Pole  
 b. Essential singularity  
 c. Removable singularity  
 d. None
15. A point at which a function  $f(z)$  ceases to be analytic is called  
 a. Zero  
 b. singularity  
 c. Pole  
 d. Limit point
16. If  $f(z)$  has a pole at  $z = a$ , then  $z \rightarrow a$   
 a.  $|f(z)| \rightarrow 0$   
 b.  $|f(z)| \rightarrow a$   
 c.  $|f(z)| \rightarrow \infty$   
 d.  $|f(z)| \rightarrow -\infty$
17. The singular point  $z = 2$  of  $f(z) = e^{\frac{1}{z-2}}$  is  
 a. pole  
 b. Essential singularity  
 c. Removable singularity  
 d. None
18. The singular point  $z = 0$  of  $f(z) = \log z$  is  
 a. Isolated  
 b. Non-isolated  
 c. Bounded  
 d. None

19. Cosiz is equal to

- a. Coshz  
c. Cosz

- b. Sinh z  
d. Sinz

20.

The value of  $\oint_C \frac{\sin^6 z}{z - \frac{\pi}{6}} dz$  where  $|z| = 1$  is

a.  $\frac{\pi i}{32}$

b.  $\frac{\pi i}{22}$

c.  $\frac{\pi i}{44}$

d. None

### ( Descriptive )

Time : 2 hrs. 30 mins.

Marks : 50

*[ Answer question no.1 & any four (4) from the rest ]*

1. Evaluate the limits

10

a.  $\lim_{z \rightarrow 2e^{i\pi/3}} \frac{z^3 + 8}{z^4 + 4z^2 + 16}$

b.  $\lim_{z \rightarrow e^{i\pi/3}} \left( z - e^{i\pi/3} \right) \left( \frac{z}{z^3 + 1} \right)$

c.  $\lim_{z \rightarrow 0} \left( \frac{\sin z}{z} \right)^{1/z^2}$

2. Evaluate

3+4+3  
=10

a.  $\oint_C \frac{\cosh(\pi z)}{z(z^2 + 1)} dz$  where  $C$  is  $|z| = 2$ .

b.  $\oint_C \frac{e^{3z}}{z - \pi i} dz$  where  $C$  is

i. the circle  $|z - 1| = 4$

ii. the ellipse  $|z - 2| + |z + 2| = 6$

c. Find the values of constants  $a, b, c, d$  such that the function

$f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$  is analytic.

3. Prove that if  $f(z)$  is continuous in a simply-connected region  $R$  and if  $\oint_C f(z) dz = 0$  around every simple closed curve  $C$  in  $R$ , then  $f(z)$  is analytic in  $R$ .

10

4. State and prove Cauchy Integral formula. Evaluate  $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{z^2 + 1} dz$  where  $|z| = 3$ .

6+4=10

5. a. Find the radius of convergence

5+5=10

(i)  $\frac{z}{2} + \frac{1.3}{2.5} z^2 + \frac{1.3.5}{2.5.8} z^3 + \dots$

(ii)  $\sum_{n=0}^{\infty} \frac{z^n}{2^n + 1}$

b. Evaluate  $\frac{1}{2\pi} \int_0^{2\pi} \sin^2 \left( \frac{\pi}{6} + 2e^{i\theta} \right) d\theta$

6. Verify Green's theorem in the plane for  $\oint_C x^2 y dx + (y^3 - xy^2) dy$  10

where  $C$  is the boundary of the region enclosed by the circles  $x^2 + y^2 = 4, x^2 + y^2 = 16$ .

7. a. Write the statement of Milne-Thomson method. Find the regular function  $f(z) = u + iv$  where 6+4=10

$$u = e^{-x} \left\{ (x^2 - y^2) \cos y + 2xy \sin y \right\}.$$

- b. Evaluate  $\oint_C \frac{e^{zt} dz}{z^2 (z^2 + 2z + 2)}$  around the circle  $C$  with  $|z| = 3$ .

8. What kind of singularities the following functions have 2×5=10

a.  $f(z) = \frac{1}{\sin z - \cos z}$  at  $z = \frac{\pi}{4}$

b.  $f(z) = \frac{\cot \pi z}{(z-a)^2}$  at  $z = 0$  and  $z = \infty$

c.  $f(z) = \frac{1 - e^z}{1 + e^z}$  at  $z = \infty$

d.  $f(z) = \tan \frac{1}{z}$  at  $z = 0$

e.  $f(z) = \operatorname{cosec} \frac{1}{z}$  at  $z = 0$

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