

**M.SC. MATHEMATICS
FIRST SEMESTER
ABSTRACT ALGEBRA I
MSM – 103**

**SET
A**

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs.

Full Marks: 35

Time: 15 mins.

(Objective)

Marks: 10

Choose the correct answer from the following:

1×10=10

1. Let $U(n)$ be the set of all positive integers less than n and relatively prime to n . Then $U(n)$ is a group under multiplication modulo n . For $n = 248$, the number of elements in $U(n)$ is
 - a. 60
 - b. 120
 - c. 180
 - d. 240
2. Which of the following is/are true
 - a. The group $\mathbb{Z}_{100}$ is Abelian but not cyclic.
 - b. The group $U(12)$ is both Abelian and cyclic.
 - c. The group $\mathbb{Z}_{100}$ is cyclic but not Abelian.
 - d. The group $U(12)$ is Abelian but not cyclic.
3. The value of α for which $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 is
 - a. 5
 - b. 15
 - c. 20
 - d. 25
4. Choose the correct statement:
 - a. The order of $\sigma = (4\ 5)(2\ 3\ 7)$ is 3 and $\tau = (1\ 4)(3\ 5\ 7\ 8)$ is 8
 - b. The order of $\sigma = (4\ 5)(2\ 3\ 7)$ is 6 and $\tau = (1\ 4)(3\ 5\ 7\ 8)$ is 8
 - c. The order of $\sigma = (4\ 5)(2\ 3\ 7)$ is 6 and $\tau = (1\ 4)(3\ 5\ 7\ 8)$ is 4
 - d. The order of $\sigma = (4\ 5)(2\ 3\ 7)$ is 2 and $\tau = (1\ 4)(3\ 5\ 7\ 8)$ is 2
5. Consider the following statements
P: The permutation $(12)(134)(152)$ is an odd permutation.
Q: The symmetric group S_7 contain an element of order 14
 - a. P is true but Q is false
 - b. P is false and Q is true
 - c. P and Q both are true
 - d. P and Q both are false
6. Consider the statements
P: A_8 contains an element of order 15.
Q: The permutation $(124)(3578)$ has order 12.
R: The maximum order of any element in S_{10} is 21.
 - a. Only Q is true
 - b. All are false
 - c. P and Q are true, but R is false
 - d. All are true

7. Let G be a non-cyclic group of order 15 and $a \in G$ be a non-identity element. Then the value of $\text{ord}(a)$ can be
 - a. 1
 - b. 5
 - c. 7
 - d. 15
8. The number elements of order 10 in a cyclic group of order 50 is
 - a. 1
 - b. 10
 - c. 4
 - d. None of these
9. If $\text{ord}(a^5) = 12$ then what is the possibility for $\text{ord}(a)$ is
 - a. 20
 - b. 60
 - c. 45
 - d. None of these
10. Let a be a generator of a cyclic group of order 40. Then which of the following element have order 40:
 - a. a^{12}
 - b. a^{15}
 - c. a^{23}
 - d. a^{20}

Descriptive

Marks : 25

1. a. Let G be an Abelian group with identity e . Prove that
$$H = \{x \in G : x^2 = e\}$$
is a subgroup of G . 3+2=5
b. Let $f = (123)(145)$. Write f^{99} in cycle form.
2. a. Show that A_5 has 24 elements of order 5, 20 elements of order 3,
and 15 elements of order 2. 4+2+4=10
b. Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.
c. If H is a subgroup of G , then prove that the set
 $\{x \in G : xh = hx, \forall h \in H\}$ is a subgroup of G .
3. a. Show that $U(10)$ is a group. Find the order of $U(10)$. Is $U(10)$
cyclic? If yes, find all the generator of $U(10)$. Also, find the order of
each element of $U(10)$. List all the subgroup of $U(10)$. 4+3+3=10
b. How many elements of order 2 and 5 are there in A_6 ?
c. Let $\beta \in S_7$ and suppose $\beta^4 = (2143567)$. Find β .

$$H = \{x \in G : x^2 = e\}$$

is a subgroup of G .

b. Let $f = (123)(145)$. Write f^{99} in cycle form.

$$4+2+4=10$$

$$4+3+3=10$$

4. a. List all the elements of $\mathbb{Z}_{40}$ that have order 10. 3+4+3
=10
 b. List all the subgroup of a cyclic group $G = \langle a \rangle$ of order 30.
 c. Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{pmatrix}$
 and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$

Write f and g as product of 2-cycles. Also, compute each of the following:

- (i) f^{-1} and g^{-1}
 (ii) fg and gf
5. a. Determine the number of elements of order 5 in $\mathbb{Z}_5 \oplus \mathbb{Z}_{25}$. 5+5=10
 b. Determine all homomorphisms from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{24}$.

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