

**M.Sc. MATHEMATICS
SECOND SEMESTER
ABSTRACT ALGEBRA II
MSM – 203
[USE OMR FOR OBJECTIVE PART]**

**SET
A**

Duration: 1:30 hrs.

Full Marks: 35

Time: 15 mins.

(Objective)

Marks: 10

Choose the correct answer from the following:

1×10=10

- Which of the following is/are true for a group G of order 103 ?
 - G has no maximal normal subgroup
 - G has exactly one maximal normal subgroup
 - G has at least one maximal normal subgroup
 - G has more than one maximal normal subgroup
- Which of the following statement is/are not necessarily true?
 - A group of order 4 is solvable
 - A group of order 12 is solvable.
 - A group of order 9 is solvable.
 - None of these
- Which of the following is/are false?
 - An abelian group always has a composition series.
 - Any group of order 4 has a composition series.
 - Any group of order 6 has a composition series.
 - None of these
- For $n \in \mathbb{N}$, let $n\mathbb{Z} = \{nk : k \in \mathbb{Z}\}$. Then the number of units of $\frac{\mathbb{Z}}{12\mathbb{Z}}$ and $\frac{\mathbb{Z}}{13\mathbb{Z}}$, respectively, are
 - 12, 13
 - 11, 12
 - 4, 11
 - 8, 11
- Which of the following is/are true?
 - $\mathbb{Z}_{12}$ is integral domain but not a field.
 - The polynomial ring $\mathbb{Z}_6[x]$ is an integral domain.
 - The ring $\mathbb{Z}[x]$ is not an integral domain
 - The ring $\mathbb{Z}[\sqrt{2}]$ is an integral domain.
- If $\mathbb{Z}[i]$ is the ring of Gaussian integers, the quotient $\frac{\mathbb{Z}[i]}{\langle 3-i \rangle}$ is isomorphic to
 - $\mathbb{Z}/10\mathbb{Z}$
 - $\mathbb{Z}/4\mathbb{Z}$
 - $\mathbb{Z}/3\mathbb{Z}$
 - $\mathbb{Z}$

7. Let F be a field with non-zero characteristic, then
- F has a subfield isomorphic to $\mathbb{Q}$.
 - F has a subfield isomorphic to $\mathbb{Z}_p$ or $\mathbb{Q}$.
 - F has a subfield isomorphic to $\mathbb{Z}_p$ for some prime p .
 - None of these
8. $\frac{\mathbb{Z}_2[x]}{\langle x^3+x^2+1 \rangle}$ is
- an integral domain but not a field.
 - a field with 8 elements.
 - a field with 9 elements
 - a field with infinitely many elements.
9. Which of the following is/are true?
- $S_n, n \geq 5$ is a solvable group but not Nilpotent
 - $S_n, n \geq 5$ is both solvable and Nilpotent group
 - $S_n, n \geq 5$ is a Nilpotent group but not solvable
 - $S_n, n \geq 5$ is neither solvable nor Nilpotent group
10. The value of $(2x + 1) + (x + 2)$ in $\mathbb{Z}_3[x]$ is
- 0
 - 1
 - $x + 2$
 - $2x + 1$

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(Descriptive)

Time : 1 hr. 15 mins.

Marks: 25

[Answer question no.1 & any two (2) from the rest]

1. Let $S = \{a + ib : a, b \in \mathbb{Z}, b \text{ is even}\}$. Show that S is a subring of $\mathbb{Z}[i]$ but not an ideal of $\mathbb{Z}[i]$. 5

2. a. Prove that - Every nilpotent group is solvable. Justify the converse of the statement. 2+3+2+3
=10
 - b. Show that the group $(\mathbb{Q}, +)$ has no maximal normal subgroup.
 - c. Find the composition series of S_3 . Prove or disprove that S_3 is solvable.
 - d. Prove that - A subgroup of solvable group is solvable.

3. a. Prove that - A non-zero finite integral domain is a field. 2×5=10
 - b. Let $R = \{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10. Prove that R is a field.
 - c. Derive the set of units in the ring of 2×2 matrices with real entries.
 - d. Prove that - For every prime p , the ring of integer modulo p , $\mathbb{Z}_p$ is a field.
 - e. Prove or disprove that $\mathbb{Z} \times \mathbb{Z}$ is not an integral domain.

4. a. Show that $\frac{\mathbb{Z}_3[x]}{\langle x^2+x+1 \rangle}$ is not a field. 1+2+2+5
=10
 - b. Show that the ideal $\langle x \rangle$ is a prime ideal but not a maximal ideal in $\mathbb{Z}[x]$.
 - c. How many elements are in $\frac{\mathbb{Z}[i]}{\langle 3+i \rangle}$. List them.
 - d. Find all ring homomorphism from the ring $\mathbb{Z}_{20}$ to $\mathbb{Z}_{30}$.

5. a. Examine which of the polynomials below are irreducible over $\mathbb{Q}$.

$$1+2+2+5=10$$

(i) $x^5 + 9x^4 + 12x^2 + 6$

(ii) $x^5 + 5x^2 + 1$

(iii) $\frac{3}{7}x^4 - \frac{2}{7}x^2 + \frac{9}{35}x + \frac{3}{5}$

- b. Show that $x^5 + 2x + 4$ is irreducible over $\mathbb{Q}$.

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