

**M.SC. MATHEMATICS
THIRD SEMESTER
FUNCTIONAL ANALYSIS
MSM – 303
[USE OMR FOR OBJECTIVE PART]**

2025/11

**SET
A**

Duration: 1:30 hrs.

Full Marks: 35

(Objective)

Time: 15 mins.

Marks: 10

1×10=10

Choose the correct answer from the following:

1. Which condition is true for Normed Linear Space

- a. $\|x + y\| \leq \|x\| + \|y\|$ b. $\|x + y\| > \|x\| + \|y\|$
c. $\|x + y\| = \|x\| + \|y\|$ d. $\|x + y\| < \|x\| + \|y\|$

2. When we can say that a norm $\|\cdot\|$ on a vector space X is equivalent to a norm $\|\cdot\|_0$ on X ?

- a. $\forall \alpha, b > 0$ such that $\forall x, y \in X, \alpha \|x\|_0 \leq \|x\| \leq b \|x\|_0$
b. $\exists \alpha, b > 0$ such that $\forall x, y \in X, \alpha \|x\|_0 \leq \|x\| \leq b \|x\|_0$
c. $\exists \alpha, b > 0$ such that $\forall x, y \in X, \alpha \|x\|_0 \leq \|x\| \leq b \|x\|_0$
d. $\exists \alpha, b > 0$ such that $\forall x, y \in X, \alpha \|x\|_0 \leq \|x\| \leq b \|x\|_0$

3. An Inner product $\langle x, y \rangle$ is a

- a. Continuous and bounded function of $X \times X$
b. Continuous function of $X \times X$
c. Continuous and derivable function of $X \times X$
d. Continuous function of $X \times Y$

4. A Normed Linear Space which is complete as a metric space is said

- a. Unitary Space b. Banach Space
c. Euclidean Space d. None of the above

5. Let $\langle \cdot, \cdot \rangle$ be an Inner product of a linear space X . Then

- a. $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle, \forall x, y, z \in X, \alpha, \beta \in K$
b. $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle, \forall x, y, z \in X, \alpha, \beta \in X$
c. $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle, \forall x, y, z \in K, \alpha, \beta \in K$
d. $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle, \forall x, y, z \in X, \alpha, \beta \in K$

6. Consider $T : R^2 \rightarrow R$, which of the following is linear map
- $T(x, y) = x^2 y$
 - $T(x, y) = xy$
 - $T(x, y) = 2x - y$
 - $T(x, y) = x^2$
7. In a Normed Linear Space every convergent sequence is a
- Only sequence
 - Convergent sequence
 - Cauchy Sequence
 - None of the above
8. Schwarz Inequality is
- $|\langle x, y \rangle|^2 \geq \langle x, x \rangle \cdot \langle y, y \rangle$
 - $|\langle x, y \rangle|^2 \leq \langle x, x \rangle \cdot \langle y, y \rangle$
 - $|\langle x, y \rangle|^2 < \langle x, x \rangle \cdot \langle y, y \rangle$
 - $|\langle x, y \rangle|^2 = \langle x, x \rangle \cdot \langle y, y \rangle$
9. Polarization Identity is known as
- $4 \langle x, y \rangle = \sum_{k=1}^4 i^k \langle x + i^k y, x + i^k y \rangle$
 - $4 \langle x^k, y \rangle = \sum_{k=1}^4 i^k \langle x + i^k y, x + i^k y \rangle$
 - $4 \langle x, y^k \rangle = \sum_{k=1}^4 i^k \langle x + i^k y, x + i^k y \rangle$
 - $4 \langle x, y \rangle = \sum_{k=1}^4 i^k \langle x + i y, x + i y \rangle$
10. The set of all bounded linear Operators from a Normed space X into a Normed space Y is a Banach Space if
- Both X and Y must be Banach
 - X is a Banach Space
 - Both X and Y must be Banach & Bounded
 - Y is a Banach Space

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(**Descriptive**)

Time : 1 hr. 15 mins.

Marks : 25

[Answer question no.1 & any two (2) from the rest]

- State and Proof Hahn Banach Theorem 2+3=5
- Prove that every Normed Linear Space is a Metric space. But the converse may not be true. Also Show that Real Linear Space and Complex Linear Space are Banach Space under the norm 2+3+5=10

$$\|x\| = |x|$$

3. Write the Definition of Continuity in Normed Linear Space and Heine's Definition of Continuity 4+6=10

What is Linear Functional. On a finite dimensional vector space

Prove that

$$\|0\| = \|0\|_0$$

4. Let $\langle, \rangle$ be an Inner product of a linear space X . Then 5+5=10

(i) Prove that every inner product space is a normed linear space with the norm induced by the inner product

(ii) Prove that $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$

5. What do you mean by continuity of Inner Product and Uniformly convex. Verify the continuity of inner product $\langle x, y \rangle$ on $X \times X$ 4+6=10

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