2025/12

SET A

B.Sc. MATHEMATICS
FIRST SEMESTER
CALCULUS
BMT – 911A IDMJ [REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

[Objective]

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. What is the value of $\lim_{x \to 0} \frac{1-\cos x}{x^2} e^x$?

a. 1 b. $-\frac{1}{2}$

c. $\frac{1}{2}$ d. 2

2. The equation of tangent at a point (x, y) on the curve $y = f(x)$ is

a. $Y + y = f'(x)(X + x)$ b. $Y - y = \frac{1}{f'(x)}(X - x)$

c. $Y - y = f'(x)(X - x)$ d. $Y - y = f(x)(X - x)$

3. If $y = \log(ax + b)$ then $y_n = ?$

a. $\frac{(-1)^{n-1} n!}{(ax+b)^n} a^n$ b. $\frac{(-1)^{n-1}(n-1)!}{(ax+b)^n} a^n$

c. $\frac{(-1)^{n-1}(n-1)!}{(ax+b)^n} a$ d. $\frac{(-1)^n (n-1)!}{(ax+b)^n} a^n$

4. Show that $f(x) = x|x|$ is

a. Derivable at origin b. Continuous at origin

c. Derivable and continuous at origin d. Analytic at origin

5. A function $f(x)$ is said to be continuous at the point $x = a$ if

a. $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x) = f(a)$

b. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$

c. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \neq f(a)$

d. $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x) \neq f(a)$

6. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}; xy \neq 0$, then $\frac{\partial u}{\partial y} = ?$

a. $\frac{\partial u}{\partial y} = x - 2y \tan^{-1}\frac{x}{y}$

b. $\frac{\partial u}{\partial y} = x + 2y \tan^{-1}\frac{x}{y}$

c. $\frac{\partial u}{\partial y} = y - 2y \tan^{-1}\frac{x}{y}$

d. $\frac{\partial u}{\partial y} = x - 2 \tan^{-1}\frac{x}{y}$

7. Radious of Curvature in Pedal Eqquation is

a. $\rho = \frac{dr}{dp}$

b. $\rho = r\frac{dr}{dp}$

c. $\rho = r^2\frac{dr}{dp}$

d. $\rho = -r\frac{dr}{dp}$

8. What is value of $\frac{d}{dx}(xc) = ?, c$ is constant

a. c

b. 1

c. $-c$

d. -1

9. $\lim_{x \to 0} \frac{a^x - b^x}{x} = ?$

a. $\log\left(\frac{b}{a}\right)$

b. $\log\left(\frac{a}{b}\right)$

c. $\log x$

d. $-\log x$

10. The curvature of a circle is
    a. A function
    b. A radious
    c. A continuous function
    d. A constant

11. If a function $f(x)$ is finitely derivable at c,then
    a. bounded
    b. $f(x)$ is also continuous at c
    c. unbounded function
    d. none of the above

12. The Asymptote of the Cissoid $y^2(a-x)=x^3$ is a line whose equation is
    a. $x=a$
    b. $y=a$
    c. $x=0$
    d. $y=0$

13. If the centre of curvature for each point on a curve be taken,we get a new curve known as
    a. Concave curve
    b. Evolute
    c. Radious of curve
    d. None of the above

14. IF $f(x)=(x-1)(x-2)(x-3), x\in[0,4]$, then $c=?$
    a. $c=6$
    b. $c=\dfrac{6\pm 2\sqrt{3}}{8}$
    c. $c=\dfrac{6+2\sqrt{3}}{8}$
    d. $c=\dfrac{6-2\sqrt{3}}{8}$

15. $\int \cos^2 x dx = ?$
    a. $\dfrac{1}{2}\left(x+\dfrac{\sin 2x}{2}\right)$
    b. $\dfrac{1}{2}\left(x+\dfrac{\cos 2x}{2}\right)$
    c. $\dfrac{1}{2}\left(\dfrac{\sin 2x}{2}\right)$
    d. $\dfrac{1}{2}\left(x-\dfrac{\sin 2x}{2}\right)$

16. What is the domain of $f(x,y)=\dfrac{1}{\log x+\log y}$
    a. $\{(x,y): 0>x, 0<y\}$
    b. $\{(x,y): 0<x, 0>y\}$
    c. $\{(x,y): 0>x, 0>y\}$
    d. $\{(x,y): 0<x, 0<y\}$

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16. What is the domain of $f(x,y)=\dfrac{1}{\log x+\log y}$
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    c. $\{(x,y):0>x,0>y\}$
    d. $\{(x,y):0<x,0<y\}$

17. $f_x(x,y) = ?$

a. $f_x(x,y) = \lim \dfrac{f(x-h,y) - f(x,y)}{h} ash \to 0$

b. $f_x(x,y) = \lim \dfrac{f(x+h,y) + f(x,y)}{h} ash \to 0$

c. $f_x(x,y) = \lim \dfrac{f(x+h,y) - f(x,y)}{h} ash \to 0$

d. $f_x(x,y) = \lim \dfrac{f(x+h,y) - f(x,y)}{-h} ash \to 0$

18. If a function f is

$[a,b]$

(i)continuous in a closed interval

$]a,b[$ $c \in]a,b[$

(ii)derivable in the open interval ,then there exist at least one point such that $f'(c) = \dfrac{f(b) - f(a)}{b-a}$.

This statement Is known as

a. Rolle's Theorem
b. Lagrange's Mean Value Theorem
c. Cauchy's Mean Value Theorem
d. Darboux's Theorem

19. The radious of curvature at any point of $s = c\tan\psi$ is

a. $c\sec^2\psi$
b. $c\sec\psi$
c. $c\tan^2\psi$
d. $c\tan\psi$

20. If $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

then $\dfrac{d^2y}{dx^2} = ?$

a. $\dfrac{\sec\theta}{a\theta}$
b. $\dfrac{\sec^2\theta}{a\theta}$
c. $\dfrac{\sec^3\theta}{\theta}$
d. $\dfrac{\sec^3\theta}{a\theta}$

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Descriptive

Time : 2 hrs. 30 min. **Marks: 50**

[Answer question no.1 & any four (4) from the rest]

1. Show that **4+6=10**

$\lim_{x \to 0+0} \frac{1}{x} = \infty$, $\lim_{x \to 0-0} \frac{1}{x} = -\infty$ and $\lim_{x \to 0} \frac{1}{x}$ does not exist .

Also Examine the continuity of the function defined by

$$f(x) = \begin{cases} -x^2, x \le 0 \\ 5x-4, 0 < x < 2 \\ 4x^2 - 3x, 1 < x < 2 \\ 3x+4, x \ge 2 \end{cases}$$ at the points $x = 0,1,2$

2. **5+5=10**

In the curve $r^m = a^m \cos m\theta$,Prove that (a) $\frac{ds}{d\theta} = a \sec^{\frac{m-1}{m}} m\theta$

(b) $a^{2m} \frac{d^2 r}{ds^2} + mr^{2m-1} = 0$

3. What do you mean by Asymptotes.Find the Asymptotes of **2+8=10**

$x^3 + 2x^2 y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$

4. **5+5=10**

Find $\frac{d^n y}{dx^n}$ if

(i) $y = \cos^4 x$

(ii) $y = e^{ax} \cos^2 x \sin x$

5. Find the reduction formula for **10**

$$\int \sin^n x dx$$

6. State and Prove that Rolle's Theorem **2+8=10**

7. Find the nth derivative of

(i) $y = \dfrac{x^2}{(x+2)(2x+3)}$

(ii) $y = x \log \dfrac{x-1}{x+1}$

8. Prove that the equation of the normal to the Astroid **10**

$$x^{2/3} + y^{2/3} = a^{2/3}$$

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