

**B.SC. MATHEMATICS
FIRST SEMESTER
ALGEBRA
BMT-MAJ-111
[USE OMR FOR OBJECTIVE PART]**

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30mins.

(Objective)

Marks: 20

Choose the correct answer from the following:

1×20=20

- If $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$ then $A \oplus B$ is
 - $\{4, 5\}$
 - $\{1, 2, 3, 6, 7\}$
 - $\{1, 2, 3\}$
 - $\{6, 7\}$
- Consider the following relation R on the set $A = \{1, 2, 3, 4\}$:
 $R = \{(1, 1), (1, 3), (2, 4), (3, 1), (3, 3), (4, 3)\}$
Then R is
 - symmetric
 - reflexive
 - transitive
 - None of these
- Let R be the following equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}$:
 $R = \{(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)\}$
The set $[2]$ is given by
 - $\{1, 5, 2\}$
 - $\{2, 3, 6\}$
 - $\{2, 4, 6\}$
 - None of these
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$ and $g(x) = x + 3$. Then $(g \circ f)(2)$ is
 - 7
 - 25
 - 4
 - 5
- Let $X = \{1, 2, 3, 4\}$. Which of the following relation below is a function from X into X ?
 - $f = \{(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)\}$
 - $g = \{(3, 1), (4, 2), (1, 1)\}$
 - $h = \{(2, 1), (3, 4), (1, 4), (4, 4)\}$
 - All the above
- Let p be "It is cold" and let "It is raining". The statement $p \vee q$ describes
 - It is cold or it is raining.
 - It is cold and raining.
 - It is cold or it is not raining.
 - None of these.
- The polar form of the complex number $-i$ is
 - $\cos 0 + i \sin 0$
 - $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$
 - $\cos \pi + i \sin \pi$
 - $\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$
- If $a = 2 + 3i, b = 3 + i$ then ab is equal to
 - $3 - 11i$
 - $-3 + 11i$
 - $-3 - 11i$
 - $3 + 11i$

9. The value of $\frac{3-2i}{-1+i}$ is
- $\frac{-5}{2} + \frac{i}{2}$
 - $\frac{-5}{2} - \frac{i}{2}$
 - $\frac{5}{2} + \frac{i}{2}$
 - $\frac{5}{2} - \frac{i}{2}$
10. The remainder of $x^5 + 5x^4 + x^3 + 5x^2 + 2x + 11$ when divided by $x + 5$ is
- 1
 - 1
 - 0
 - None of these
11. The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is
- Symmetric matrix
 - Skew-symmetric matrix
 - Neither symmetric nor Skew-symmetric
 - Both symmetric and Skew-symmetric
12. The trace of the matrix is $A = \begin{bmatrix} 2 & -5 & 7 \\ 3 & 7 & 9 \\ 11 & 8 & -9 \end{bmatrix}$ is
- 5
 - 2
 - 0
 - 11
13. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 2 & 4 & 6 \end{bmatrix}$ is
- 0
 - 1
 - 2
 - 3
14. The following system of equations is:
- $$\begin{aligned} x - 2y &= 5 \\ -2x + 4y &= 6 \end{aligned}$$
- Inconsistent
 - Consistent and have a unique solution
 - Consistent and have infinite solutions
 - The only solution is $x = y = 0$
15. The following system of equations is:
- $$\begin{aligned} 2x - 5y &= 0 \\ 6x - 15y &= 0 \end{aligned}$$
- Inconsistent
 - Consistent and have only trivial solution.
 - Consistent and have non-trivial solutions.
 - None of these
16. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^t$, then
- $x + y = 5$
 - $x = y$
 - $x = 0, y = 5$
 - None of these
17. Consider the statements:
 $p: \forall x \in \mathbb{R}, |x| \neq 0$
 $q: \forall x \in \mathbb{R}, x^2 \geq x$
 Which of the following is true?

- a. p is true, q is false
 b. p is false, q is true
 c. Both p and q are false
 d. Both p and q are true
18. The negation of the following statement is:
 "For all positive integers n , $n + 2 > 8$ "
- a. There exists a positive integer n such that $n + 2 \neq 8$.
 b. There exists a positive integer n such that $n + 2 > 8$.
 c. For all positive integers n , $n + 2 < 8$
 d. None of these
19. Let $A = \{2, 3, 5\}$ and let $p(x)$ be the sentence " x is prime". Then the statement "At least one number in A is prime" can be denoted as:
 a. $\forall x \in A, p(x)$
 b. $\exists x \in A, p(x)$
 c. $\exists x \in A, \neg p(x)$
 d. None of these
20. If p and q are two statements then the statement $\sim(p \wedge q)$ is equivalent to
 a. $\sim p \wedge \sim q$
 b. $\sim p \vee \sim q$
 c. $\sim p \wedge q$
 d. $\sim p \vee q$

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(**Descriptive**)

Time : 2hrs. 30mins.

Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Consider the following data for 120 mathematics students: 10
- 65 study Khasi
 45 study Assamese
 42 study Bangali
 20 study Khasi and Assamese
 25 study Khasi and Bangali
 15 study Assamese and Bangali
 8 study all three languages
- a. Find the number of students studying at least one of three languages.
 b. Find the number of students studying exactly one language.
 c. Find the number of students studying exactly two languages.
 d. Find the number of students studying none of these three languages.
 e. Find the number of students studying only Khasi.
2. a. Find the truth table of $\sim p \wedge q$. 2+3+5
 b. Show that the proposition $p \vee \sim (p \wedge q)$. =10

c. Show that $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$.

3. a. Using the method of synthetic division, find $f(x+2)$ if $f(x) = x^4 - x^3 + 2x^2 + 6x - 2$ 3+3+4=10

b. If $x^3 + 3px + q$ has a factor of the form $(x - a)^2$ show that $q^2 + 4p^3 = 0$.

c. Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B :

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$

(i) Determine the matrix of the relation.

(ii) Draw the arrow diagram of R .

(iii) Find the inverse relation R^{-1} of R .

(iv) Determine the domain and range of R .

4. a. Show that the following system of equations is inconsistent: 6+4=10

$$2x - y + z = 4$$

$$3x - y + z = 6$$

$$4x - y + 2z = 7$$

$$-x + y - z = 9$$

b. Find the inverse of the matrix

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

5. a. Find the eigenvalues of the following matrix: 5+5=10

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -6 & 1 & 3 \\ -12 & -2 & 8 \end{bmatrix}$$

b. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 4 & -2 & 2 \\ 1 & 2 & -3 & 0 \\ 3 & 6 & -4 & 3 \end{bmatrix}$$

6. a. Solve the equation $x^4 + x^2 - 2x + 6 = 0$, it is given that $1 + i$ is a root. 3+2+3+2=10

b. Apply Descartes' rule of signs to examine the nature of the roots of the equation: $x^4 + 2x^2 + 3x - 1 = 0$.

c. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and let R be the relation on A defined by " x is a multiple of y ".

(i) Write R as a set of ordered pairs.

(ii) Draw its directed graph.

(ii) Find the inverse relation R^{-1} of R . Can R^{-1} be described in words?

d. Let $A = \{1, 2, 3\}$, $B = \{a, b, c\}$, $C = \{x, y, z\}$. Consider the following relation R from A to B and relation S from B to C :

$$R = \{(1, b), (2, a), (2, c)\}$$

$$S = \{(a, y), (b, x), (c, y), (c, z)\}$$

(i) Find the composition relation $S \circ R$.

(ii) Draw arrow diagram of $S \circ R$.

7. a. Find the polar form of: $-1 - i$

3+3+4

b. Find the cube root of 1.

=10

c. If n be an integer, prove that

$$(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}.$$

8. a. If $z = a + ib$, $a, b \in \mathbb{R}$ be any complex number then define $|z|$ i.e. the modulus of z . Show that $|z|^2 = z\bar{z}$, $\bar{z}$ being the conjugate of z . Also, show that

7+3=10

$$(i) \quad \overline{z + w} = \bar{z} + \bar{w}$$

$$(ii) \quad \overline{zw} = \bar{z} \cdot \bar{w}$$

Where z and w are any two complex numbers.

b. Let $A = \{1, 2, 3, 4\}$. Consider the following relation R, S on A :

$$R = \{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$$

$$\text{and } S = A \times A$$

(i) Draw the directed graphs of R and S .

(ii) Is R reflexive, symmetric, transitive and antisymmetric?

(iii) Is S reflexive, symmetric, transitive and antisymmetric?

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