

**B.S.C. MATHEMATICS
FIRST SEMESTER
CALCULUS
BMT – 101 OLD COURSE [REPEAT]
[USE OMR FOR OBJECTIVE PART]**

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

(Objective)

Marks: 20

1×20=20

Choose the correct answer from the following:

- The number of possible functions that can be defined from $X = \{a, b\}$ to $Y = \{p, q, r\}$ is
 - 8
 - 9
 - 5
 - None of these
- The number of one-one functions from $X = \{a, b\}$ to $Y = \{p, q, r\}$ is
 - 4
 - 5
 - 6
 - 7
- The step function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer $\leq x$ is
 - Continuous for all $x \in \mathbb{R}$
 - Continuous at integers in $\mathbb{R}$
 - Discontinuous at all integers in $\mathbb{R}$
 - Discontinuous at all non-integers in $\mathbb{R}$
- The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{|x|}{x}$ is
 - Continuous at $x = 0$
 - Discontinuous at $x = 0$
 - Discontinuous at $x \neq 0$
 - False for all the cases here
- Let $y = f(x) = |x|$, $x \in \mathbb{R}$. Then $x = 0$, $f(x)$ is
 - Both continuous and derivable
 - Continuous but not derivable
 - Derivable but not continuous
 - Neither continuous nor derivable
- If $y = x \sin x$ then $\frac{d^2y}{dx^2}$ at $x = 0$ is
 - 2
 - 2
 - 0
 - 1
- If $f(x) = \tan x$, then $f'(x)$ at $x = \frac{\pi}{4}$ is
 - 2
 - $-\sqrt{2}$
 - 2
 - 2
- If $y = \sin x$, $x \in \mathbb{R}$, then y_n is
 - $\cos\left(\frac{n\pi}{2} + x\right)$
 - $\sin\left(\frac{\pi}{2} + x\right)$
 - $\cos\left(\frac{n\pi}{2} - x\right)$
 - $\sin\left(\frac{n\pi}{2} - x\right)$

9. The gradient of the tangent to the curve $y = x^3 - 2x$ at $x = 1$ is
 a. 0
 b. 1
 c. -1
 d. 2
10. The gradient of the normal to the curve $y = f(x)$ at a point x_0 on the curve is given by
 a. $f'(x_0)$
 b. $\frac{1}{f'(x_0)}$
 c. $-\frac{1}{f'(x_0)}$
 d. None of these
11. Let $y = f(x)$ be a function of real numbers such that
 (i) $f(x)$ is continuous in $[a, b]$
 (ii) $f(x)$ is derivable in (a, b)
 (iii) $f(a) = f(b)$
 Then by Rolle's theorem
 a. $f'(x) > 0$ for all $x \in [a, b]$
 b. $f'(x) < 0$ for all $x \in [a, b]$
 c. $f'(x) = 0$ for at least one $x \in (a, b)$
 d. None of these
12. The formula $\frac{y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}}$ denotes the length of
 a. Tangent
 b. Normal
 c. Subtangent
 d. Subnormal at a point of the curve $y = f(x)$
13. If a function $y = f(x)$ is
 (i) Continuous in $[a, b]$
 (ii) Derivable in (a, b)
 Then there is a point c such that $f'(c) = \frac{f(b) - f(a)}{b - a}$ where
 a. $a \leq c < b$
 b. $a < c \leq b$
 c. $a \leq c \leq b$
 d. $a < c < b$
14. If $y = f(x) = \log x$, then y_n is given by
 a. $\frac{1}{x^n}$
 b. $\frac{(n-1)!}{x^n}$
 c. $\frac{n!}{x^n}$
 d. $(-1)^{n-1} \frac{(n-1)!}{x^n}$
15. The value of y_n , where $y = (ax + b)^n$ is
 a. na^n
 b. $n! a^n$
 c. nab^n
 d. $n! b^n$
16. Rolle's theorem is valid for the function $f(x)$ if
 a. $f(x) = \tan x$ in $[0, \pi]$
 b. $f(x) = \cos \frac{1}{x}$ in $[-1, 1]$
 c. $f(x) = x^2$ in $[2, 3]$
 d. $f(x) = x(x+3)$ in $[-3, 0]$
17. The necessary condition for a function $f(x)$ to have an extreme value at $x = e$ is
 a. $f'(e) > 0$
 b. $f'(e) = 0$
 c. $f'(e) < 0$
 d. None of these

18. A function $f(x)$ has maximum value at $x = e$ if
- $f'(c) = 0$ and $f''(c) > 0$
 - $f'(c) = 0$ and $f''(c) < 0$
 - $f'(c) = 0$ and $f''(c) \neq 0$
 - $f'(c) \neq 0$ and $f''(c) = 0$
19. If $f(x) = |x|$ then
- $f'(0) = 0$
 - $f(x)$ is maximum at $x = 0$
 - $f(x)$ is minimum at $x = 0$
 - None of these
20. The function $f(x) = x^3 - 6x^2 + 24x + 4$ has
- A maximum value at $x = 2$
 - A minimum value at $x = 2$
 - A maximum value at $x = 4$ and a minimum value at $x = 6$
 - Neither maximum nor minimum at any point.

(Descriptive)

Time : 2 hrs. 30 min.

Marks: 50

[Answer question no.1 & any four (4) from the rest]

- a. Define derivative $f'(a)$ of a function $f(x)$ at a point $x = a$. Find $f'(a)$ for the function $f(x) = |x|$, if it exists. 5+5=10

b. If $y = f(x) = x^2 \sin x$ then compute $\left[\frac{dy}{dx} \right]_{x=0}$
- a. Construct all possible functions from $A = \{a, b, c\}$ to $B = \{x, y\}$. 5+5=10
Hence find out

 - the number of onto functions
 - the number of functions which are not onto.

b. Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = [x]$ where $[x]$ denotes the greatest integer $\leq x$.
Show that f is continuous at $x = 0$ from the right but it is not continuous from the left.
- a. What do you mean by $\lim_{x \rightarrow a} f(x) = l$. Show that $\lim_{x \rightarrow 0} \sin x = 0$. 5+5=10

b. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = |x| + |x - 1|$, $x \in \mathbb{R}$.
Draw the graph of the function $f(x)$ for $-1 \leq x \leq 2$.

4. a. Examine the limit of the function $f(x)$ as $x \rightarrow 2$ where

5+5=10

$$f(x) = \begin{cases} \frac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

Examine the continuity of $f(x)$ at $x = 2$.

- b. Evaluate the following limits

(i) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$

(ii) $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2}\right)^x$

5. a. Examine the continuity of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

5+5=10

$$f(x) = \begin{cases} x, & 0 \leq x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ 1-x, & \frac{1}{2} < x < 1 \end{cases}$$

- b. If $y = \left(\frac{1}{x}\right)^x$, show that $y_2(1) = 0$ where

$$y_2(1) = \left[\frac{d^2 y}{dx^2} \right]_{x=1}$$

6. a. If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$.

5+5=10

Hence by using Leibnitz theorem show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

- b. If a function $y = f(x)$ is derivable at a point x_0 in its domain of definition, then prove that it is continuous at the point.

7. a. Find the equation of tangent to the parabola $y^2 = 4ax$ at $(a, -2a)$.

5+5=10

- b. State and prove Lagrange's mean value theorem.

8. a. Investigate the maximum and minimum values of the function

5+5=10

$$f(x) = 2x^3 - 15x^2 + 36x + 10$$

- b. Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$

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