

**B.SC. MATHEMATICS
THIRD SEMESTER
REAL ANALYSIS
BMT – 301**
[USE OMR FOR OBJECTIVE PART]

2025/11

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

Marks: 20

1X20=20

(Objective)

Choose the correct answer from the following:

- The set $\mathbb{R}$ of all real numbers is
 - archimedean
 - ordered
 - field
 - all the above
- In $\mathbb{R}$,
 - every open interval is an open set.
 - every open set is an open interval.
 - no open interval is an open set.
 - no open set in an open interval.
- The limit point of $S = \{-3, 0, 1\}$ is
 - 3
 - 0
 - 1
 - None of these.
- If $A = (0, 1)$, then A' , the derived set of A is

a.	$(0, 1]$	b.	$[0, 1]$
c.	$[0, 1]$	d.	$\{0, 1\}$
- If $\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$ then $\mathbb{N}'$ is

a.	$\mathbb{N}$	b.	ϕ
c.	$\mathbb{Z}$	d.	$\mathbb{Q}$
- The sequence $\{s_n\} = \{1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, \dots\}$ is
 - bounded and convergent
 - bounded but not convergent
 - unbounded and convergent
 - neither bounded nor convergent
- The $\text{Sup}s_n$, where $s_n = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots\}$ is

a.	$\frac{1}{2}$	b.	$\frac{2}{3}$
c.	$\frac{3}{4}$	d.	1
- The $\text{Inf}s_n$, where $s_n = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\}$ is

a.	-1	b.	0
c.	$\frac{1}{2}$	d.	$-\frac{1}{2}$

9. The sequence $\left\{1, \frac{5}{2}, \frac{5}{3}, \frac{9}{4}, \frac{9}{5}, \frac{13}{6}, \frac{13}{7}, \dots\right\}$ converges to
 a. 1
 b. 2
 c. 3
 d. None of these
10. $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots \infty$ converges if
 a. $p < 1$
 b. $p < 0$
 c. $p > 1$
 d. $p = 1$
11. Let $\{s_n\}$ be a convergent sequence. Then $\{s_n\}$ converges to
 a. two distinct limits
 b. more than two distinct limits
 c. infinitely many limits
 d. a unique limit
12. Let $\sum u_n$ and $\sum v_n$ be two series of positive terms such that $\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = l, l \neq 0$. Then
 a. $\sum u_n$ converges if $\sum v_n$ diverges
 b. $\sum v_n$ converges if $\sum u_n$ diverges
 c. $\sum u_n$ and $\sum v_n$ both converge or diverge together
 d. None of the above is true.
13. Let $\sum u_n$ be a series of positive terms such that $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$. Then $\sum u_n$ converges if
 a. $l < 1$
 b. $l > 1$
 c. $l = 1$
 d. None of these happens
14. Let f be a function defined and bounded on $[a, b]$. If $L(P, f)$ and $U(P, f)$ are respectively the lower and upper Riemann sums of f over a partition P of $[a, b]$ then
 a. $L(P, f) < U(P, f)$
 b. $L(P, f) \leq U(P, f)$
 c. $L(P, f) > U(P, f)$
 d. $L(P, f) \geq U(P, f)$
15. Let $\sum u_n$ be a series of positive terms such that $\lim_{n \rightarrow \infty} u_n^{\frac{1}{n}} = l$. Then $\sum u_n$ converges if
 a. $l < 1$
 b. $l > 1$
 c. $l = 1$
 d. None of these happens
16. For any real function f defined and bounded on $[a, b]$. The lower Riemann integral $\int_a^b f(x) dx$ is defined by
 a. $\sup_p U(P, f)$
 b. $\sup_p L(P, f)$
 c. $\inf_p U(P, f)$
 d. $\inf_p L(P, f)$
17. For any real function f defined and bounded on $[a, b]$. The upper Riemann integral $\int_a^b f(x) dx$ is defined by
 a. $\sup_p L(P, f)$
 b. $\inf_p L(P, f)$
 c. $\inf_p U(P, f)$
 d. $\sup_p U(P, f)$
18. The proper integral $\int_0^1 \frac{dx}{x}$
 a. converges to 0
 b. converges to 1
 c. converges to -1
 d. diverges at $x = 0$
19. The improper integral $\int_0^2 \frac{dx}{\sqrt{x}}$
 a. converges to $\sqrt{2}$
 b. converges to $2\sqrt{2}$
 c. converges to 2
 d. diverges at $x = 0$

20. A real function f defined and bounded on $[a, b]$ will be Riemann integrable if

- a. $\int_a^b f \, dx \leq \int_a^b f \, dx$ b. $\int_a^b f \, dx \geq \int_a^b f \, dx$
 c. $\int_a^b f \, dx = \int_a^b f \, dx$ d. None of these happens.

(Descriptive)

Time : 2 hrs. 30 min.

Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. a. When is a set $A \subseteq \mathbb{R}$ said to be bounded? Define supremum and infimum of a bounded set. Is the set $A = \left\{2, \frac{1}{2}, \frac{4}{3}, \frac{3}{4}, \frac{6}{5}, \frac{5}{6}, \dots\right\}$ bounded? Find $\text{Sup } A$ and $\text{Inf } A$ if these exist. 5+1+4
=10
 b. What is a limit point of a set $A \subseteq \mathbb{R}$? Find the 'derived set' for each of the following sets:
 (i) $A = \{-3, 0, 1, 5\}$
 (ii) $B = \left\{\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \dots\right\}$
 (iii) $\mathbb{Q} = \left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$
 (iv) $C = \{x \in \mathbb{R}, 1 < x < 2\}$
2. a. When is a set $A \subseteq \mathbb{R}$ called open? Show that every open interval in $\mathbb{R}$ is an open set. Give example to show that an open set in $\mathbb{R}$ need not be an open interval. 5+5=10
 b. How do you define a bounded sequence $\{s_n\}$? Is the sequence $\{s_n\}$ given by $s_n = 1 + (-1)^{n-1} \frac{1}{n}$ bounded? If so, find its bounds.
3. a. What do you mean by the convergence of a sequence $\{s_n\}$ of real numbers? Prove that if a sequence converges, then it is bounded. 5+2+3
=10
 b. Find the n th terms for the following sequences and examine their convergence by evaluating $\lim_{n \rightarrow \infty} s_n$, where s_n is the n th term of the sequence:
 (i) $\left\{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \dots\right\}$

$$(ii) \left\{ 2, \frac{1}{2}, \frac{4}{3}, \frac{3}{4}, \frac{6}{5}, \frac{5}{6}, \dots \right\}$$

4. a. When is an infinite series $\sum u_n = u_1 + u_2 + u_3 + \dots \infty$ said to be convergent? Show that the series

5+5=10

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots$$

is convergent and it converges to the sum 1.

- b. Show that the geometric series

$$\sum ar^{n-1} = a + ar + ar^2 + ar^3 + \dots \infty$$

converges for $|r| < 1$ and diverges for $|r| \geq 1$.

5. a. Use the comparison test for the convergence of the series

5+5=10

$$\frac{1 \cdot 2}{3^2 \cdot 4^2} + \frac{3 \cdot 4}{5^2 \cdot 6^2} + \frac{5 \cdot 6}{7^2 \cdot 8^2} + \dots$$

- b. State D'Alemberts' Ratio test for convergence of an infinite series $\sum u_n = u_1 + u_2 + u_3 + \dots \infty$. Apply this test to examine the convergence of $\sum \frac{n^2-1}{n^2+1} x^n, x > 0$.

6. a. Let f be a bounded function on $[a, b], a < b$. What do you mean by upper sum $U(P, f)$ and lower sum $L(P, f)$ of f for a partition P of $[a, b]$? Hence define Riemann upper integral $\int_a^b f dx$ and Riemann lower integral $\int_a^b f dx$.

3+2+5
=10

- b. Find $\int_a^b f dx$ and $\int_a^b f dx$ if $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$

7. a. Let f be a real function defined and bounded on $[a, b]$ except at $x = a$. How do you define $\int_a^b f(x) dx$. Hence discuss the convergence of the improper integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

2+4+4
=10

- b. Evaluate the improper integral $\int_1^2 \frac{xdx}{\sqrt{x-1}}$

8. a. Show that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if $n < 1$.

5+5=10

- b. Apply comparison test for convergence of improper integrals to test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$

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