

**B.SC. MATHEMATICS
SECOND SEMESTER
TRIGONOMETRY
BMT – 202**

[USE OMR FOR OBJECTIVE PART]

**SET
A**

Duration: 1:30 hrs.

Full Marks: 35

Time: 15 mins.

(Objective)

Marks: 10

Choose the correct answer from the following:

1×10=10

- For real number θ if $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ then n is
 - an integer
 - a negative fraction
 - a positive fraction
 - an irrational number
- $\cos \frac{\theta}{5} + i \sin \frac{\theta}{5}$ is
 - equal to $(\cos \theta + i \sin \theta)^{\frac{1}{5}}$
 - Equal to $(\cos \theta + i \sin \theta)^5$
 - one of values of $(\cos \theta + i \sin \theta)^{\frac{1}{5}}$
 - one of values of $(\cos \theta + i \sin \theta)^{-\frac{1}{5}}$
- Values of $i^{\frac{1}{3}}$ are
 - $\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}, \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}, \sin \frac{9\pi}{6} + i \cos \frac{9\pi}{6}$
 - $\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}, \cos \frac{5\pi}{6} - i \sin \frac{5\pi}{6}, \cos \frac{9\pi}{6} - i \sin \frac{9\pi}{6}$
 - $\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}, \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}, \cos \frac{9\pi}{6} + i \sin \frac{9\pi}{6}$
 - $\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}, \sin \frac{5\pi}{6} + i \cos \frac{5\pi}{6}, \sin \frac{9\pi}{6} + i \cos \frac{9\pi}{6}$
- The value of $\text{Log}(a + ib), a, b \in \mathbb{R}$ is
 - $\log(a^2 + b^2) - i(\theta + 2k\pi)$
 - $\frac{1}{2} \log(a^2 + b^2) + i(\theta + 2k\pi)$
 - $\frac{1}{2} \log(a^2 + b^2) + i(\theta + k\pi)$
 - None of these
- If $z = x + iy$ then
 - $|\sin y| \leq |\sin z| \leq |\cosh y|$
 - $|\sinh y| \geq |\sin z| \geq |\cosh y|$
 - $|\sinh y| > |\sin z| > |\cosh y|$
 - $|\sinh y| < |\sin z| < |\cosh y|$
- If x, y are real, the value of $\sin(x + iy)$ is
 - $\sin x \cos y + i \cos x \sin y$
 - $\sin x \cosh y + i \cos x \sinh y$
 - $\sinh x \cos y + i \cosh x \sin y$
 - $\sinh x \cosh y + i \cos x \sin y$

7. If x, y are real, the value of $\cosh(x + iy)$ is
- a. $\cos x \cos y + i \sinh x \sinh y$ b. $\cos x \cosh y + i \sin x \sinh y$
 c. $\cosh x \cos y - i \sinh x \sin y$ d. None of these
8. The value of $\frac{\pi}{4}$ is
- a. $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots$ b. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
 c. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$ d. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$
9. If x is real or complex, e^x is defined as
- a. $\cos x - i \sin x$ b. $\sin x + i \cos x$
 c. $\sin x - i \cos x$ d. $\cos x + i \sin x$
10. The value of $\tan^{-1} \sqrt{3}$ is
- a. $\sqrt{3} + \frac{(\sqrt{3})^3}{3} + \frac{(\sqrt{3})^5}{5} + \frac{(\sqrt{3})^7}{7} + \dots$
 b. $\sqrt{2} + \frac{(\sqrt{2})^3}{3} + \frac{(\sqrt{2})^5}{5} + \frac{(\sqrt{2})^7}{7} + \dots$
 c. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
 d. $\sqrt{3} - \frac{(\sqrt{3})^3}{3} + \frac{(\sqrt{3})^5}{5} - \frac{(\sqrt{3})^7}{7} + \dots$

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(Descriptive)

Marks: 25

Time : 1 hr. 15 mins.

[Answer question no.1 & any two (2) from the rest]

1. State De Moivre's theorem and prove it for any power n integral or rational. 1+4=5

2. a. If $x \neq 1$, then solve the equation 5+5=10

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$$

- b. If n be an integer, prove that

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}$$

3. a. If $2 \cos \theta = x + \frac{1}{x}$, where θ is real, prove that $2 \cos n\theta = x^n + \frac{1}{x^n}$, 5+5=10
 n being an integer.

- b. Prove that for any real number θ ,

$$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$$

4. a. Use De Moivre's theorem to expand $\cos n\theta$ and $\sin n\theta$ in powers of $\cos \theta$ and $\sin \theta$. Hence expand $\cos x$ and $\sin x$ in ascending powers of x . 5+5=10

- b. If $\tan^{-1}(x+iy) = \alpha + i\beta$ where x, y, α, β are reals and $(x, y) \neq (0, \pm 1)$, then prove that

$$x^2 + y^2 + 2x \cot 2\alpha = 1.$$

5. a. For any real number θ prove Greory's series 5+5=10

$$\theta = \tan \theta - \frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta - \dots$$

State the condition to be satisfied by θ for validity of the series.
 Hence deduce the series for $\tan^{-1} x$ where x is a real number lying between -1 and 1 .

- b. Prove that
$$\frac{\pi}{8} = \frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} + \dots$$

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