

**B.SC. MATHEMATICS
SECOND SEMESTER
ANALYTICAL GEOMETRY
BMT – 201**
[USE OMR FOR OBJECTIVE PART]

**SET
A**

Duration: 3 hrs.

Full Marks: 70

(**Objective**)

Time: 30 min.

Marks: 20

1×20=20

Choose the correct answer from the following:

- The formula of transformation from one pair of rectangular axes to another in two dimensions with the same origin is
 a. $x = x' \sin \theta - y' \cos \theta$ b. $x = x' \sin \theta + y' \cos \theta$
 c. $x = x' \cos \theta - y' \sin \theta$ d. $x = x' \cos \theta + y' \sin \theta$
- The transformed equation of the line $y = x$ when the axes are rotated through an angle 45° is
 a. $x = 0$ b. $x = 1$
 c. $y = 1$ d. $y = 0$
- Angle between the pair of lines given by the equation $ax^2 + 2hxy + by^2 = 0$ is
 a. $\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a + b}$ b. $\theta = \tan^{-1} \frac{2\sqrt{h^2 + ab}}{a + b}$
 c. $\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a - b}$ d. $\theta = \tan^{-1} \frac{2\sqrt{h^2 + ab}}{a + b}$
- Which of the following is a non central conic
 a. Circle b. Parabola
 c. Ellipse d. None of these
- The polar equation of a conic with a focus as the pole is given by
 a. $\frac{l}{r} = 1 + e \sin \theta$ b. $\frac{l}{r} = 1 + e \tan \theta$
 c. $\frac{l}{r} = 1 + e \cos \theta$ d. None of these
- Bisector of angles between the pair of lines $ax^2 + 2hxy + by^2 = 0$
 a. $\frac{x^2 + y^2}{a - b} = \frac{xy}{h}$ b. $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$
 c. $\frac{x^2 + y^2}{a + b} = \frac{xy}{h}$ d. $\frac{x^2 + y^2}{a - b} = \frac{-xy}{h}$
- For which value of θ the term xy vanishes in the expression $ax^2 + 2hxy + by^2$
 a. $\theta = \frac{1}{2} \tan^{-1} \frac{2h}{a - b}$ b. $\theta = 2 \tan^{-1} \frac{2h}{a - b}$
 c. $\theta = \frac{1}{2} \tan^{-1} \frac{2h}{a + b}$ d. $\theta = 2 \tan^{-1} \frac{2h}{a + b}$

8. If two planes $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ are parallel then
- $aa' + bb' + cc' = 0$
 - $aa' + bb' + cc' = 0$
 - $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$
 - $\frac{a}{a'} = \frac{b}{b'} = \frac{-c}{c'}$
9. The equation of the plane through the point $(2, 3, 5)$ and parallel to the plane $2x - 4y + 3z = 9$
- $3x - 4y + 3z = 9$
 - $2x - 4y + 3z = 6$
 - $2x - 4y + 7z = 9$
 - $-2x + 4y - 3z = -7$
10. The angle between the two planes $2x + 4y - 5z + 1 = 0$ and $-x - 2y - 2z + 6 = 0$
- 90°
 - 30°
 - 0°
 - 45°
11. The necessary and sufficient condition for the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ to represent two planes is
- $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
 - $abc - 2fgh - af^2 - bg^2 = 0$
 - $abc - 2fgh - af^2 - bg^2 - ch^2 = 0$
 - $abc + 2fgh - af^2 - bg^2 = 0$
12. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is
- 10 units
 - 5 units
 - 0 unit
 - None of these
13. The centre of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is
- $(0, 0, 0)$
 - $(-u, -v, -w)$
 - (u, v, w)
 - None of these
14. The equation of an ellipsoid if the z -plane cuts the surface in the ellipse is
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 - $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
 - $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$
 - $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$
15. In the hyperboloid of 2-sheets, the section of the surface by a plane parallel to the x -plane is an ellipse given by
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} - 1$
 - $\frac{y^2}{b^2} + \frac{z^2}{c^2} + \frac{k^2}{a^2} = 1$
 - $\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} - 1$
 - $\frac{y^2}{b^2} + \frac{z^2}{c^2} + \frac{k^2}{a^2} = 0$
16. The equation of a plane in normal form is
- $x \cos \alpha - y \cos \beta + z \cos \gamma = p$
 - $x \cos \alpha + y \cos \beta + z \cos \gamma = p$
 - $x \cos \alpha + y \cos \beta - z \cos \gamma = p$
 - None of these
17. The transformed parallel axes through the new origin $(2, -1)$, the equation of the curve $2x^2 + 3y^2 - 8x + 6y = 0$ is
- $2x^2 + 3y^2 - 11 = 0$
 - $2x^2 + 3y^2 + 11 = 0$
 - $2x^2 + 3y^2 - 1 = 0$
 - $2x^2 + 3y^2 + 1 = 0$

18. Which of the following coordinates lie on the plane $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z}{4}$
- a. $(3, -1, 0)$ b. $(2, 3, 4)$
 c. $(-3, 1, 0)$ d. $(-2, -3, -4)$
19. The general second degree equation $ax^2 + 2hxy + by^2 + gx + 2fy + c = 0$ represents an ellipse if
- a. $ab - h^2 = 0$ b. $ab - h^2 < 0$
 c. $ab - h^2 > 0$ d. None of these
20. The equation of the line AB passing through the points $A(a, 0, 0)$ and $B(0, a, 0)$ is
- a. $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ b. $\frac{x}{a} = \frac{y}{b} = \frac{z-a}{a}$
 c. $\frac{x-a}{a} = \frac{y}{a} = \frac{z}{0}$ d. $\frac{x-a}{a} = \frac{y}{-a} = \frac{z}{0}$

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(Descriptive)

Time : 2 hrs. 30 min.

Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. If by the rotation of rectangular axes, the equation $17x^2 + 18xy - 7y^2 = 1$ reduces to the form $Ax^2 + By^2 = 1$, find A and B . Also find the angle through which the axes are rotated. 10
2. Reduce the equation $9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$ to the standard form. 10
3. If the straight lines represented by the equation $x^2(\tan^2 \theta + \cos^2 \theta) - 2xy \tan \theta + y^2 \sin^2 \theta = 0$ make angles α, β with the axis of x . Show that $\tan \alpha - \tan \beta = 2$. 10
4. Find all the asymptotes of the curve $x^2 + y^2 - 3axy = 0$. 10
5. Show that the locus of the middle points of a system of parallel chords of an ellipse is a straight line passing through the origin. 10

6. A square $ABCD$ of diagonal $2a$ is folded along the diagonal AC so that the planes DAC, BAC are at right angles. Show that the shortest distance between DC and AB is $\frac{2a}{\sqrt{3}}$. 10
7. Find the length and equation of the line of the shortest distance between the lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$. 10
8. Prove that the equation of the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$. If $2d$ is the shortest distance between the given lines, then prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$. 10

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