

**B.SC. MATHEMATICS
FOURTH SEMESTER
COMPLEX ANALYSIS
BMT - 941 IDMJ [REPEAT]
[USE OMR FOR OBJECTIVE PART]**

2025/06

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

Marks: 20

(Objective)

Choose the correct answer from the following:

1×20=20

1. The argument of the complex number $z = \frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is
 - a. $\frac{\pi}{6}$
 - b. $\frac{\pi}{4}$
 - c. $\frac{\pi}{2}$
 - d. None
2. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+1}$ is
 - a. $\frac{\pi}{6}$
 - b. $\frac{\pi}{4}$
 - c. $\frac{\pi}{3}$
 - d. None
3. If $z = 1 + i\sqrt{3}$ then $|\arg z| + |\arg \bar{z}|$ equals
 - a. $\frac{\pi}{3}$
 - b. $\frac{2\pi}{3}$
 - c. 0
 - d. $\frac{\pi}{2}$
4. (z_1, z_2) and (z_3, z_4) are two pairs of conjugate complex numbers, then $\arg \frac{z_1}{z_3} + \arg \frac{z_2}{z_4}$ is equal to
 - a. 0
 - b. $\frac{\pi}{2}$
 - c. π
 - d. $-\frac{\pi}{2}$
5. The number of solutions of the equation $z^2 = \bar{z}$ is
 - a. 2
 - b. 3
 - c. 4
 - d. None
6. The number of solutions of $z^2 + \bar{z}z = 0$ is
 - a. 4
 - b. 3
 - c. 2
 - d. 5
7. The solution of the equation $|z| - z = 1 + 2i$ is
 - a. $\frac{3}{2} - 2i$
 - b. $\frac{3}{2} + 2i$
 - c. $2 - \frac{3}{2}i$
 - d. None

8. For any complex number z , the minimum value of $|z| + |z - 1|$ is
 a. 1
 b. 0
 c. $\frac{1}{2}$
 d. $\frac{3}{2}$
9. The maximum value of $|z|$ when z satisfies the condition $\left|z - \frac{2}{z}\right| = 2$ is
 a. $\sqrt{3} - 1$
 b. $\sqrt{3}$
 c. $\sqrt{3} + 1$
 d. $\sqrt{2} + \sqrt{3}$
10. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ then
 a. $x = 3, y = 1$
 b. $x = 1, y = 3$
 c. $x = 0, y = 3$
 d. $x = 0, y = 0$
11. An analytic function with constant modulus
 a. is variable
 b. is constant
 c. is zero
 d. does not exist
12. Function satisfying Laplace's equation are known as
 a. regular
 b. homomorphic
 c. harmonic
 d. conjugate
13. Function $f(z) = xy + iy$ is
 a. everywhere continuous and analytic
 b. everywhere continuous but not analytic
 c. discontinuous but analytic everywhere
 d. neither continuous nor analytic
14. For a function $f(z) = u + iv$ to be analytic, Cauchy-Riemann equation
 a. $u_x = v_y, u_y = -v_x$
 b. $u_x = -v_y, u_y = v_x$
 c. $u_x = v_y, u_y = v_x$
 d. $u_x = -v_y, u_y = -v_x$
15. Function $w = z^{1/2}$ is
 a. single valued
 b. multiple valued
 c. both
 d. None
16. $\text{Cosech } z$ is equal to
 a. $\text{Cosh } z$
 b. $\text{Sinh } z$
 c. $\text{Cos } z$
 d. $\text{Sin } z$
17. The value of $\lim_{z \rightarrow 1} \frac{\sqrt{z^2 + 3} - 2}{z - 1}$ is
 a. 1
 b. 0
 c. $\frac{1}{2}$
 d. $-\frac{1}{2}$
18. The value of $\lim_{z \rightarrow 0} \frac{z}{z}$ is
 a. 0
 b. 1
 c. does not exist
 d. None

19. The modulus of $z = 3i$ is
 a. 3
 b. -3
 c. 0
 d. None
20. The radius of the circle $|\sqrt{5} + 2i|$ is
 a. 9
 b. -9
 c. 3
 d. -3

-- --- --

(Descriptive)

Time : 2 hrs. 30 min.

Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. Evaluate the limits 5+5=10
- a. $\lim_{z \rightarrow 2e^{i\pi/3}} \frac{z^3 + 8}{z^4 + 4z^2 + 16}$
- b. $\lim_{z \rightarrow e^{i\pi/3}} \left(z - e^{i\pi/3} \right) \left(\frac{z}{z^3 + 1} \right)$
2. a. Prove that $f(z) = \frac{z}{z^4 + 1}$ is continuous at all points inside and on the unit circle $|z| = 1$, except at four points. Also determines the four points. 5+5=10
- b. For what values of z are the functions continuous
- i. $f(z) = \frac{3z}{z^2 + 4}$
- ii. $f(z) = \frac{z^2 + 1}{z^2 - 3z + 2}$
3. Prove that in polar form Cauchy-Riemann equation can be written as $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$. 10

4. Prove that the functions are harmonic and find its harmonic conjugate and corresponding analytic function in terms of z . 5+5=10
- a. $u = x^3 - 3xy^2$
- b. $u = \frac{1}{2} \log(x^2 + y^2)$
5. a. Verify Cauchy-Riemann equation for the function $f(z) = e^{z^2}$. 5+5=10
b. Prove that analytic function with constant modulus is constant.
6. Examine whether the functions are uniformly continuous 5+5=10
- a. $f(z) = \frac{1}{z}$
- b. $f(z) = 3z - 2$
7. If z_1 and z_2 both satisfy the relation $z + \bar{z} = 2|z - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$ then find the imaginary part of $(z_1 + z_2)$. 10
8. Evaluate the limits $2\frac{1}{2} \times$
4 = 10
- a. $\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$
- b. $\lim_{z \rightarrow i} \frac{z^2+1}{z^6+1}$
- c. $\lim_{z \rightarrow e^{i\pi/4}} \frac{z^2}{3z^4-2z^3+8z^2-2z+5}$
- d. $\lim_{z \rightarrow i} \frac{z^2}{z-i}$

== *** ==