

**B.SC. MATHEMATICS
FOURTH SEMESTER
COMPLEX ANALYSIS
BMT - 941 IDMJ**
[USE OMR FOR OBJECTIVE PART]

2025/06

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

(Objective)

Marks: 20

Choose the correct answer from the following:

$1 \times 20 = 20$

1. The argument of the complex number $z = \frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is
 - a. $\frac{\pi}{6}$
 - b. $\frac{\pi}{4}$
 - c. $\frac{\pi}{2}$
 - d. None
2. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+1}$ is
 - a. $\frac{\pi}{6}$
 - b. $\frac{\pi}{4}$
 - c. $\frac{\pi}{3}$
 - d. None
3. If $z = 1 + i\sqrt{3}$ then $|\arg z| + |\arg \bar{z}|$ equals
 - a. $\frac{\pi}{3}$
 - b. $\frac{2\pi}{3}$
 - c. 0
 - d. $\frac{\pi}{2}$
4. (z_1, z_2) and (z_3, z_4) are two pairs of conjugate complex numbers, then $\arg \frac{z_1}{z_3} + \arg \frac{z_2}{z_4}$ is equal to
 - a. 0
 - b. $\frac{\pi}{2}$
 - c. π
 - d. $-\frac{\pi}{2}$
5. The number of solutions of the equation $z^2 = \bar{z}$ is
 - a. 2
 - b. 3
 - c. 4
 - d. None
6. The number of solutions of $z^2 + \bar{z}z = 0$ is
 - a. 4
 - b. 3
 - c. 2
 - d. 5
7. The solution of the equation $|z| - z = 1 + 2i$ is
 - a. $\frac{3}{2} - 2i$
 - b. $\frac{3}{2} + 2i$
 - c. $2 - \frac{3}{2}i$
 - d. None

8. For any complex number z , the minimum value of $|z| + |z - 1|$ is
- 1
 - 0
 - $\frac{1}{2}$
 - $\frac{3}{2}$
9. The maximum value of $|z|$ when z satisfies the condition $\left|z - \frac{2}{z}\right| = 2$ is
- $\sqrt{3} - 1$
 - $\sqrt{3}$
 - $\sqrt{3} + 1$
 - $\sqrt{2} + \sqrt{3}$
10. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ then
- $x = 3, y = 1$
 - $x = 1, y = 3$
 - $x = 0, y = 3$
 - $x = 0, y = 0$
11. An analytic function with constant modulus
- is variable
 - is constant
 - is zero
 - does not exist
12. Function satisfying Laplace's equation are known as
- regular
 - homomorphic
 - harmonic
 - conjugate
13. Function $f(z) = xy + iy$ is
- everywhere continuous and analytic
 - everywhere continuous but not analytic
 - discontinuous but analytic everywhere
 - neither continuous nor analytic
14. For a function $f(z) = u + iv$ to be analytic, Cauchy-Riemann equation
- $u_x = v_y, u_y = -v_x$
 - $u_x = -v_y, u_y = v_x$
 - $u_x = v_y, u_y = v_x$
 - $u_x = -v_y, u_y = -v_x$
15. Function $w = z^{1/2}$ is
- single valued
 - multiple valued
 - both
 - None
16. Cosiz is equal to
- Coshz
 - Sinhz
 - Cosz
 - Sinz
17. The value of $\lim_{z \rightarrow 1} \frac{\sqrt{z^2+3}-2}{z-1}$ is
- 1
 - 0
 - $\frac{1}{2}$
 - $-\frac{1}{2}$
18. The value of $\lim_{z \rightarrow 0} \frac{z}{z}$ is
- 0
 - 1
 - does not exist
 - None

19. The modulus of $z = 3i$ is
 a. 3
 b. -3
 c. 0
 d. None
20. The radius of the circle $|\sqrt{5} + 2i|$ is
 a. 9
 b. -9
 c. 3
 d. -3

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(Descriptive)

Time : 2 hrs. 30 min.

Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. Evaluate the limits 5+5=10

a. $\lim_{z \rightarrow 2e^{i\pi/3}} \frac{z^3 + 8}{z^4 + 4z^2 + 16}$

b. $\lim_{z \rightarrow e^{i\pi/3}} \left(z - e^{i\pi/3} \right) \left(\frac{z}{z^3 + 1} \right)$

2. a. Prove that $f(z) = \frac{z}{z^4 + 1}$ is continuous at all points inside and on the unit circle $|z| = 1$, except at four points. Also determines the four points. 5+5=10

b. For what values of z are the functions continuous

i. $f(z) = \frac{3z}{z^2 + 4}$

ii. $f(z) = \frac{z^2 + 1}{z^2 - 3z + 2}$

3. Prove that in polar form Cauchy-Riemann equation can be written as $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$. 10

4. Prove that the functions are harmonic and find its harmonic conjugate and corresponding analytic function in terms of z . 5+5=10

a. $u = x^3 - 3xy^2$

b. $u = \frac{1}{2} \log (x^2 + y^2)$

5. a. Verify Cauchy-Riemann equation for the function $f(z) = e^{z^2}$. 5+5=10
b. Prove that analytic function with constant modulus is constant.

6. Examine whether the functions are uniformly continuous 5+5=10

a. $f(z) = \frac{1}{z}$

b. $f(z) = 3z - 2$

7. If z_1 and z_2 both satisfy the relation $z + \bar{z} = 2|z - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$ then find the imaginary part of $(z_1 + z_2)$. 10

8. Evaluate the limits $2\frac{1}{2} \times$
4 = 10

a. $\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$

b. $\lim_{z \rightarrow i} \frac{z^2+1}{z^6+1}$

c. $\lim_{z \rightarrow e^{i\pi/4}} \frac{z^2}{z^4+z+1}$

d. $\lim_{z \rightarrow i} \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$

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