REV-01
MSP/17/22

2024/12

SET A

M.Sc. PHYSICS
FIRST SEMESTER
QUANTUM MECHANICS
MSP – 101

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. The probability density $|\Psi(x,t)|^2$ is defined as
   - a. $\Psi(x,t)\Psi(x,t)$
   - b. $\Psi^*(x,t)\Psi^*(x,t)$
   - c. $\Psi^*(x,t)\Psi(x,t)$
   - d. none of these

2. The operator representation of momentum in quantum mechanics is
   - a. $-i\hbar\frac{\partial}{\partial x}$
   - b. $-i\frac{\partial}{\partial x}$
   - c. $-\frac{\partial}{\partial x}$
   - d. $-\hbar\frac{\partial}{\partial x}$

3. A wave function has the form, $\psi(x,t) = Ae^{-ibt/\hbar}$ (A, b are real, independent of both $x$and t). We can conclude that
   - a. The probability density is zero
   - b. The probability density oscillates with time
   - c. The probability density is a constant over time
   - d. The probability density decays with time

4. Heisenberg's uncertainty relation between x and p is (symbols have their usual meanings)
   - a. $\sigma_x\sigma_p \geq \hbar/2$
   - b. $\sigma_x\sigma_p < \hbar/2$
   - c. $\sigma_x\sigma_p = 0$
   - d. $\sigma_x\sigma_p = \infty$

5. The energy difference between adjacent simple harmonic oscillator (1D) energy levels is
   - a. $\frac{1}{2}\hbar\omega$
   - b. $\hbar\omega$
   - c. $\frac{3}{2}\hbar\omega$
   - d. $2\hbar\omega$

6. The commutator of x and p, $[x,p] =$
   - a. i
   - b. $\hbar$
   - c. 1
   - d. $i\hbar$

7. The number of nodes in the first excited state of the harmonic oscillator potential is
   - a. 0
   - b. 1
   - c. 2
   - d. 3

8. $a_+$is the raising operator and ψ_0 is the ground state for the simple harmonic oscillator. The correct choice is

a. $a_+\psi_0 = A\psi_{-1}$ b. $a_+\psi_0 = A\psi_1$
c. $a_+\psi_0 = A\psi_0$ d. $a_+\psi_0 = 0$

9. The energy in the nth stationary state in an infinite square well potential varies with $n$as

a. $\frac{1}{n}$ b. n
c. $1/n^2$ d. n^2

10. The infinite square well potential has the form $V(x) = \begin{cases} 0, \text{if } 0 \le x \le a \\ \infty, \text{otherwise} \end{cases}$

The probability of finding the particle within the range $-\infty \le x \le -a$ is

a. 0 b. 0.25
c. 0.75 d. 1

11. If $\Psi(t,x) = Ae^{-i(\omega t - kx)}$, then the eigenvalue of $\frac{d}{dt}$will be?

a. $i\,\omega$ b. $-i\,\omega$
c. ω d. $-\,\omega$

12. If $\Psi(t,x) = Ae^{-i(\omega t - kx)}$, then the eigenvalue of $\frac{d}{dx}$will be?

a. $i\,k$ b. $-i\,k$
c. k d. -k

13. If ω is the angular frequency of a wave, then it will be

a. $\omega = E\,h$ b. $\omega = E/h$
c. $\omega = h/E$ d. $\omega = \frac{1}{Eh}$

14. The momentum operator in 3D will be

a. $\vec{p} = i\,h\vec{\nabla}$ b. $\vec{p} = -i\,h\vec{\nabla}$
c. $\vec{p} = -\frac{i}{h}\vec{\nabla}$ d. $\vec{p} = \frac{i}{h}\vec{\nabla}$

15. The determinant of the metric tensor $g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix}$ will be

a. $r^2\sin^2\theta$ b. $r^4\sin^2\theta$
c. $r^2/\sin^2\theta$ d. $r^4/\sin^2\theta$

16. The de-Alembertian operator in 3D will be

a. $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2$ b. $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \vec{\nabla}$
c. $\Box = -\frac{\partial^2}{\partial t^2} + \frac{1}{c^2}\vec{\nabla}$ d. $\Box = -\frac{\partial^2}{\partial t^2} + \vec{\nabla}$

17. The arc length in cylindrical system in 3D will be

a. $ds^2 = dr^2 + r^{-2}d\varphi^2 + dz^2$ b. $ds^2 = dr^2 + r^2 d\varphi^2 + dz^2$
c. $ds^2 = dr^2 + r\,d\varphi^2 + dz^2$ d. $ds^2 = dr^2 + r^2 d\varphi^2 + r^2 dz^2$

18. The scale factor $h_{ij} = \sqrt{g_{ij}}$ in cylindrical system will be

a. $h_r = r, h_\varphi = r, h_z = 1$ b. $h_r = 1, h_\varphi = r, h_z = 1$

c. $h_r = 1, h_\varphi = 1, h_z = r$

d. $h_r = r, h_\varphi = 1, h_z = 1$

19. The Klein-Gordon equation is given by

a. $\partial_\mu \partial^\mu \Psi = \left(\frac{m_0 c}{\hbar}\right)^2 \Psi$

b. $\partial_\mu \partial^\mu \Psi = \left(\frac{m_0 c}{\hbar}\right) \Psi$

c. $\partial^\nu \partial^\mu \Psi = \left(\frac{m_0 c}{\hbar}\right) \Psi$

d. $\partial^\nu \partial^\mu \Psi = \left(\frac{m_0 c}{\hbar}\right)^2 \Psi$

20. The energy operator will be

a. $E = -i\hbar \frac{\partial}{\partial t}$

b. $E = i\hbar \frac{\partial}{\partial t}$

c. $E = -\frac{i}{\hbar}\frac{\partial}{\partial t}$

d. $E = \frac{i}{\hbar}\frac{\partial}{\partial t}$

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Descriptive

Time : 2 hrs. 30 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. Draw the first three stationary states of the infinite square well (bounded between 0 to a). Identify the number of nodes in each of them. **3+2+5 =10**

b. A particle in the infinite square well has as its initial wave function an even mixture of the first two stationary states:

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

(i) Normalize $\Psi(x,0)$. [No explicit integration is allowed.]

(ii) Find $\Psi(x,t)$and $|\Psi(x,t)|^2$. (Express the latter in terms of sinusoidal functions of time, eliminating the exponentials with the help of Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$.) Let $\omega \equiv \pi^2\hbar/2ma^2$.

2. a. Derive covariant form of the Klein-Gordon equation using the relativistic energy-momentum relation. **6+4=10**

b. Obtain the equation of continuity using the Klein-Gordon equation.

3. For the nth stationary state of the harmonic oscillator, compute the followings using the ladder operators. **2+2+3+ 3=10**

(a) $\langle x \rangle$

(b) $\langle p \rangle$

(c) $\langle x^2 \rangle$

(d) $\langle p^2 \rangle$

[Helpful: $a_+\psi_n = \sqrt{n+1}\psi_{n+1}$, $a_-\psi_n = \sqrt{n}\psi_{n-1}$,

The ladder operators are defined as $a_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$.]

4. For the given line-element $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ 4+6=10

(a) Write down the metric tensor $g_{\mu\nu}$ and find its determinant.

(b) Find inverse metric tensor $g^{\mu\nu}$ for the above line-element.

5. A free particle, which is initially localized in the range $-a < x < a$, is released at time $t = 0$: 2+3+2+3=10

$$\Psi(x,0) = \begin{cases} A & \text{if } -a < x < a \\ 0 & \text{otherwise,} \end{cases}$$

Where A and a are positive real constants.

(a) Normalize $\Psi(x,0)$.

(b) Find $\phi(k)$ using the relation $\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Psi(x,0)e^{-ikx}dx$.

(c) Construct $\Psi(x,t)$ following the relation

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)e^{-i\left(kx-\frac{\hbar k^2}{2m}t\right)}dk.$$

[Note that you cannot solve it analytically. This could be done numerically. So just concentrate on the construction of the wave function.]

(d) Discuss the limiting cases for $\Psi(x,0)$ and $\phi(k)$ (a very large, and a very small).

6. **a.** Derive the Dirac equation in covariant form. 4+6=10

b. Discuss all the properties associated with α's and β's matrices.

7. The needle of a broken car speedometer is free to swing, and bounces perfectly off the pins at either end, so that if you give it a flick it is equally likely to come to rest at any angle between 0 to π. 1+2+2+3+2=10

(a) What is the probability density, $\rho(\theta)$?

(b) Plot $\rho(\theta)$ as a function of θ, from $\frac{-\pi}{2}$ to $\frac{3\pi}{2}$.

(c) Compute $\langle\theta\rangle, \langle\theta^2\rangle$, and $\langle\sigma\rangle$ for this distribution.

8. **a.** Write the differences between the Klein-Gordon equation and the Dirac equation. 6+4=10

b. why the Klein-Gordon equation possesses negative energy? Explain it?

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