M.Sc. PHYSICS
FIRST SEMESTER
STATISTICAL MECHANICS
MSP – 102 [REPEAT]
[USE OMR FOR OBJECTIVE PART]

SET
A

Duration: 3 hrs.

Full Marks: 70

(**Objective**)

Time: 30 min.

Marks: 20

Choose the correct answer from the following: *1×20=20*

1. The partition function for a 3 dimensional monoatomic gas is given by____?

    a. $\frac{1}{h^2}\iint e^{-\beta/kT}d^3q\,d^3p$

    b. $\frac{1}{h}\iint e^{-\beta E}d^3q\,d^3p$

    c. $\frac{1}{h^3}\iint e^{-\beta E}dq\,dp$

    d. $\frac{1}{h^3}\iint e^{-E/kT}d^3q\,d^3p$

2. In canonical ensemble, the relative r.m.s fluctuation in energy is

    a. $\frac{\sqrt{kT^2}}{U}$

    b. $\frac{\sqrt{kTC_v}}{U}$

    c. $\sqrt{\frac{kT^2C_v}{U}}$

    d. $\frac{\sqrt{kT^2C_v}}{U}$

3. Which of the following equation describes the Liouville's theorem?

    a. $[\rho, H] + \frac{d\rho}{dt} = \frac{\partial\rho}{\partial t}$

    b. $\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} - [\rho, H]$

    c. $\frac{d\rho}{dt} = [\rho, H] + \frac{\partial\rho}{\partial t}$

    d. $\frac{d\rho}{dt} = [\rho, H] - \frac{\partial\rho}{\partial t}$

4. The partition function of a single harmonic oscillator is

    a. $\frac{1}{\beta h}\sqrt{\frac{m}{k}}$

    b. $[\beta h\omega]^{-1/2}$

    c. $[\beta h\omega]^{-N}$

    d. $\frac{1}{\beta h}\sqrt{\frac{k}{m}}$

5. In Hamilton's equations of motion, what does H represent?
    a. Entropy
    b. Heat
    c. Total energy
    d. Helmholtz free energy

6. The Helmholtz free energy of an one dimensional classical harmonic oscillator is
    a. $Nk_BT/[\beta h\omega]$
    b. $Nk_BTln[\beta h\omega]$
    c. $Nk_BTln[\beta h\omega]^{-N}$
    d. $Nk_BT[\beta h\omega]^{-2}$

7. The probability density of a system of particles in a canonical ensemble is proportional to..... ?
   a. $g(E)e^{\beta E}$
   b. $e^{-\beta E}$
   c. $g(E)e^{-\beta E}$
   d. $g(E)e^{-\beta E/kT}$

8. Which of the following is the dispersion relation in the energy fluctuation in canonical ensemble?
   a. $\sigma = \sqrt{k_B T^2 C_v}$
   b. $\sigma = \sqrt{k_B T^2 C_p}$
   c. $\sigma = \sqrt{k_B/T^2 C_v}$
   d. $\sigma = \sqrt{k_B T C_v}$

9. The internal energy of a 1D harmonic oscillator is given by
   a. $U = N\beta^{-2}$
   b. $U = N\beta^{-1}$
   c. $U = N\beta$
   d. $U = \beta^{-1}$

10. In statistical mechanics, what does the concept of hypervolume represent in phase space?
    a. The total energy of the system in hyperspace
    b. The volume of microstates accessible to the system in phase space
    c. The volume of the region in phase space that corresponds to the system's energy range
    d. The entropy of the system in phase space

11. Which of the following is true about the trace of a density matrix?
    a. It is always 0
    b. It is always 1
    c. It is equal to the number of dimensions
    d. It depends on the state of the system

12. Which statistical ensemble is used to describe a system in thermal equilibrium with a heat reservoir?
    a. Microcanonical ensemble
    b. Canonical ensemble
    c. Grand canonical ensemble
    d. None of the above

13. What phenomenon occurs at very low temperatures in a system of bosons?
    a. Phase transition to a liquid
    b. Classical behavior emerges
    c. Fermi energy increase
    d. Bose-Einstein condensation

14. Fermi Dirac statistics cannot be applied to
    a. Protons
    b. Electrons
    c. Photons
    d. Fermions

15. In canonical ensembles the individual system are separated by
    a. Rigid, permeable, conducting walls
    b. Rigid, impermeable, conducting walls
    c. Rigid, impermeable, non-conducting walls
    d. None of the above

16. The canonical density matrix can be written as (symbols have usual meaning)
    a. $\rho_{nn} = e^{+\beta E_n}/z$
    b. $\rho_{nn} = e^{-\beta(E_i - \mu N_i)}/z$
    c. $\rho_{nn} = e^{-\beta E_n}/z$
    d. $\rho_{nn} = e^{\beta(E_n - n_i)}/z$

17. The sum of all the microscopic form of energy is called
    a. Total energy
    b. Phase energy
    c. System energy
    d. Internal energy

18. The difference between Bosons and Fermions is that Bosons wave function is
    a. Continuous
    b. single valued
    c. symmetric
    d. antisymmetric

19. Which of the following is a fundamental postulate of quantum mechanics that applies to quantum statistical mechanics?
    a. Particles can be classified as classical or quantum
    b. The state of a system is represented by a wave function or a density matrix
    c. Energy is always conserved in a closed system
    d. All systems eventually reach thermal equilibrium

20. Which of the following conditions applies to the density matrix of a quantum system?
    a. It must be a unitary matrix
    b. It must be Hermitian and have a trace of 1
    c. It must only have real eigenvalues
    d. It must be diagonalizable

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(Descriptive)

Time : 2 hrs. 30 min.   Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Derive the Maxwell Boltzmann velocity distribution law and show its variation with temperature.   5+5=10
    b. Derive the Bose-Einstein distribution law.

2. a. What is a partition function? Write down its expression for distinguishable particles.   1+1=2
    b. Explain the concept of energy fluctuations in a canonical ensemble in correspondence with the micro-canonical ensemble.   8

3. a. State and derive the Liouville's theorem and explain its consequences.   6+4=10
    b. Find out the partition function of classical ideal gas using canonical ensemble approach.

4. a. Elaborate the concept of micro-canonical, canonical and grand canonical ensemble. 4+6=10

   b. Find out the partition function of a classical harmonic oscillator and deduce its corresponding thermodynamic variables.

5. a. Explain the energy fluctuation in a canonical ensemble using the case of classical ideal gas. 6+4=10

   b. Consider a pure state, $|\psi(\theta)\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle + e^{i\theta}|\downarrow\rangle\right]$. Express the density matrix in terms of a 2×2 matrix.

6. a. Write the postulates of quantum statistical mechanics. 4+3+3 =10

   b. Prove that the expectation value of the mixed state is a convex sum of expectation values of its constituent pure state, i.e.

   $$\langle A\rangle_{\rho_{mix}} = \sum_{i} P_i\langle\psi_i|A|\psi_i\rangle$$

   c. Explain quantum mechanical canonical ensemble.

7. a. What is black body radiation? Give the properties of photon gas and derive the Planck's radiation law it in terms of wavelength. 2+8=10

8. a. Write the properties of Fermions. Derive the expression for the density of states of Fermions in three dimensions and discuss its physical significance. 2+8=10

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